

Flavored leptogenesis with quasi degenerate neutrinos in a broken cyclic symmetric model

Biswajit Adhikary^{a,*}, Mainak Chakraborty^{b,†}, Ambar Ghosal^{b,‡}

a)Department of Physics, Gurudas College, Narkeldanga, Kolkata-700054, India

b) Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India

May 25, 2016

Abstract

Cyclic symmetry in the neutrino sector with the type-I seesaw mechanism in the mass basis of charged leptons and right chiral neutrinos (N_{iR} , $i = e, \mu, \tau$) generates two fold degenerate light neutrino and three fold degenerate heavy neutrino mass spectrum. Consequently, such scheme, produces vanishing one light neutrino mass squared difference and lepton asymmetry. To circumvent such unphysical outcome, we break cyclic symmetry in the diagonal right chiral neutrino mass term by a small breaking parameter. Nonzero mass squared differences and mixing angles are generated with the help of the small breaking parameter. Smallness of the breaking parameter opens up a possibility of resonant leptogenesis. Assuming complex Yukawa couplings, we derive generalized expressions flavor dependent CP asymmetry parameters (ε_i^α) which are valid for quasi degenerate as well as hierarchical mass spectrum of right handed neutrinos. There after we set up the chain of coupled Boltzmann equations (which are flavor dependent too) which have to be solved in order to get the final lepton asymmetries. Depending upon the temperature regime the CP asymmetries and the Boltzmann equations may also be flavor independent. As our goal is to study the enhancement of CP asymmetry due to quasi degeneracy of right handed neutrinos, we select only the lowest allowed (by neutrino oscillation data) value of breaking parameter (and other corresponding Lagrangian parameters) and estimate the baryon asymmetry parameter Y_B . Experimental constraint of Y_B introduces a bound on right handed neutrino mass which remained unrestricted by neutrino oscillation data.

*biswajitadhikary@gmail.com

†mainak.chakraborty@saha.ac.in

‡ambar.ghosal@saha.ac.in

1 Introduction

Many experimental observations suggest the excess of matter over antimatter in the universe. In fact, no evidence of appreciable amount of antimatter has been found yet. Various considerations indicate that the universe has started its evolution from a baryon symmetric state and the baryon asymmetry observed in the present era is generated dynamically. The process responsible for the generation of baryon asymmetry is known as Baryogenesis [1–5]. There are three necessary conditions known as Sakharov conditions [6] which have to be satisfied in order to generate baryon asymmetry dynamically. They are (i) Baryon number violation, (ii) C and CP violation, (iii) departure from thermal equilibrium. The baryon asymmetry of the universe is expressed popularly by two nearly equivalent parameters η_B and Y_B , mathematically which can be written as

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} \quad (1.1)$$

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} \quad (1.2)$$

where $n_B, n_{\bar{B}}, n_\gamma$ are number densities of baryons, antibaryons, photons respectively and s is the entropy density. After the recent result of Planck satellite experiment, the value of η_B ¹ can vary mostly within the range as $(6.02 - 6.18) \times 10^{-10}$ [7–9]. The lower limit arises solely due to the analysis of the Planck data at 68% limit whereas inclusion of gravitational lensing data with the above shifts the value of η_B to the higher end.

Among the various existing mechanisms to generate baryon asymmetry at electroweak scale, baryogenesis via leptogenesis [10–13] is a simple and attractive mechanism. In this mechanism lepton asymmetry generated at a high scale ($\sim 10^9$ GeV) gets converted into baryon asymmetry (η_B) at electroweak scale due to $B + L$ violating sphaleron interactions [14, 15]. Within the framework of Standard $SU(2)_L \times U(1)_Y$ Model (SM) with at least two right chiral neutrinos (N_{iR}) there is a Dirac type Yukawa interaction of N_{iR} with electroweak leptons and Higgs doublet. At a high scale where $SU(2)_L \times U(1)_Y$ is unbroken, N_{iR} 's with definite mass can decay into both (i) charged lepton with charged scalar ($N_{iR} \rightarrow e_\alpha^-, \phi^+$) and (ii) light neutrino with neutral scalar ($N_{iR} \rightarrow \nu_\alpha, \phi^0$). CP conjugate decays of the above processes are also admitted due to Majorana property of N_{iR} . If out of equilibrium decay of N_{iR} in conjugate process occurs at different rate than the actual process, a net lepton number asymmetry will be realized, which then gets converted into baryon asymmetry due to sphaleronic interactions of the SM.

In the present work, we investigate the interrelation between leptogenesis, heavy right chiral neutrinos and flavor mixing of light neutrinos. In fact, we first constrain the parameter space

¹The value of η_B and Y_B at present epoch are related as $\eta_B = 7.04 Y_B$.

utilizing extant neutrino oscillation data [16–18], and subsequently we further restrict the parameter space incorporating the reported value of baryon asymmetry by Planck satellite experiment. In particular, we consider a well defined model based on $SU(2)_L \times U(1)_Y$ gauge symmetry with three right chiral neutrinos $N_{e_R}, N_{\mu_R}, N_{\tau_R}$ invoking type-I seesaw mechanism and discrete cyclic symmetry. The model has already been investigated by the authors recently in the context of an application of the general methodology developed to calculate three mass eigenvalues, three mixing angles and Dirac and Majorana phases of a general complex 3×3 Majorana neutrino mass matrix. In this work, we study baryogenesis via leptogenesis in detail.

We briefly describe the model here. The cyclic symmetry considered as follows

$$\begin{aligned} \nu_{e_L} &\rightarrow \nu_{\mu_L} \rightarrow \nu_{\tau_L} \rightarrow \nu_{e_L}, \\ N_{e_R} &\rightarrow N_{\mu_R} \rightarrow N_{\tau_R} \rightarrow N_{e_R}. \end{aligned} \tag{1.3}$$

The symmetry invariant neutrino mass matrix can generate nonzero θ_{13} and other two mixing angles within the experimentally constrained range at the leading order. In spite of having those attractive properties the effective neutrino mass matrix encounters a serious problem of degenerate eigenvalues which is strictly forbidden by the neutrino oscillation experimental data. Due to such degeneracy in eigenvalues the mixing angles can not be determined uniquely. To overcome those shortcomings the cyclic symmetry is broken in the right chiral neutrino sector only and the effective neutrino mass matrix is constructed again with this broken symmetric right handed neutrino mass matrix and symmetry preserving Dirac neutrino mass matrix. The eigenvalues and mixing angles of this broken symmetric effective neutrino mass matrix are calculated directly (without any perturbative approach) using the generalized formulas [19].

The second part of the work deals with generation of baryon asymmetry through the production of lepton asymmetry. In our symmetry breaking scheme the breaking parameter is taken to be small and hence the masses of the three right handed neutrinos are not far apart from each other. Therefore, instead of hierarchical leptogenesis here we have to use the resonant leptogenesis formalism. Again we know that the lepton flavors (e, μ, τ) involved in the process may or may not be separately distinguishable depending upon the temperature regime in which we are working, therefore, the study of leptogenesis is done in three different regimes, viz **(i) fully flavored** ($m(\text{GeV}) < 10^9$): three lepton flavors (e, μ, τ) are completely distinguishable, **(ii) τ -flavored** ($10^9 < m(\text{GeV}) < 10^{12}$): we can't differentiate between e and μ but τ is distinguishable, **(iii) un-flavored** ($m(\text{GeV}) > 10^{12}$): all three flavors act indistinguishably. At first the expressions of flavor dependent CP asymmetry parameters are obtained for resonant leptogenesis formalism (CP asymmetry parameters required for the other two cases ((ii) and (iii)) can be obtained by summing over the flavor indices). These CP asymmetry parameters are then inserted into Boltzmann equations

which have to be solved to get the final value of the lepton asymmetry. This lepton asymmetry will be converted into baryon asymmetry through sphaleron process. The CP asymmetry parameters and several decay and scattering terms of the Boltzmann equation involve Lagrangian parameters which are already constrained by neutrino oscillation data. Our parametrization of the neutrino mass matrix is such that the right handed neutrino mass remains unrestricted by the oscillation data. Calculation of baryon asymmetry parameter (η_B or Y_B) requires mass of the right handed neutrino along with other restricted set of parameters including the phases. The experimental bound on η_B introduces a limit on the mass of the right handed neutrino and the signs of the phase parameters also get fixed.

We organize the present work as follows: In Section 2 we briefly discuss the model under consideration. Starting from a most general leptonic mass term we have generated the effective neutrino mass matrix(m_ν) through type-I seesaw mechanism. Parametrization and diagonalization of the broken symmetric mass matrix is also described in brief in this section. Different subsections of Section 3 deals with the detailed mathematical expressions of flavor dependent CP asymmetry parameters and chain of coupled Boltzmann equations which are solved to obtain the flavor dependent/independent lepton asymmetry. Section 4 contains the recipe to get the baryon asymmetry from lepton asymmetry in different energy regimes. Outcome of the numerical analysis for various cases are presented in Section 5. Finally we summarize the whole analysis in Section 6.

2 Cyclic symmetric model

The most general leptonic Yukawa terms of the Lagrangian in the present model is

$$-\mathcal{L}_{\text{mass}} = (m_\ell)_{ll'} \bar{l}_L l'_R + m_{D_{ll'}} \bar{\nu}_{lL} N_{l'R} + M_{R_{ll'}} \bar{N}_{lL}^c N_{l'R} \quad (2.1)$$

where $l, l' = e, \mu, \tau$. We demand that the neutrino part of the Lagrangian is invariant under the cyclic permutation [19–23] as given in eq.(1.3). The cyclic symmetric Dirac neutrino mass matrix m_D takes the form

$$m_D = \begin{pmatrix} y_1 & y_2 & y_3 \\ y_3 & y_1 & y_2 \\ y_2 & y_3 & y_1 \end{pmatrix} \quad (2.2)$$

where in general all the entries are complex. The matrix m_D can be written in terms of Yukawa couplings as $(m_D)_{ij} = h_{ij}^\nu \frac{v}{\sqrt{2}}$, where h_{ij}^ν are the Yukawa couplings and v is the VEV ($v = 246$ GeV). We assume a basis in which the right handed neutrino mass matrix (M_R) and charged lepton mass matrix (m_ℓ) are mass diagonal. Further, imposition of cyclic symmetry dictates the texture of M_R

as

$$M_R = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix}. \quad (2.3)$$

Invoking type-I seesaw mechanism the effective neutrino mass matrix m_ν ,

$$m_\nu = -m_D M_R^{-1} m_D^T \quad (2.4)$$

takes the form as

$$m_\nu = -\frac{1}{m} \begin{pmatrix} y_1^2 + y_2^2 + y_3^2 & y_1 y_2 + y_2 y_3 + y_3 y_1 & y_1 y_2 + y_2 y_3 + y_3 y_1 \\ y_1 y_2 + y_2 y_3 + y_3 y_1 & y_1^2 + y_2^2 + y_3^2 & y_1 y_2 + y_2 y_3 + y_3 y_1 \\ y_1 y_2 + y_2 y_3 + y_3 y_1 & y_1 y_2 + y_2 y_3 + y_3 y_1 & y_1^2 + y_2^2 + y_3^2 \end{pmatrix}. \quad (2.5)$$

With a suitable choice of parameters m_ν can be rewritten as

$$m_\nu = m_0 \begin{pmatrix} 1 + p^2 e^{2i\alpha} + q^2 e^{2i\beta} & p e^{i\alpha} + q e^{i\beta} + p q e^{i(\alpha+\beta)} & p e^{i\alpha} + q e^{i\beta} + p q e^{i(\alpha+\beta)} \\ p e^{i\alpha} + q e^{i\beta} + p q e^{i(\alpha+\beta)} & 1 + p^2 e^{2i\alpha} + q^2 e^{2i\beta} & p e^{i\alpha} + q e^{i\beta} + p q e^{i(\alpha+\beta)} \\ p e^{i\alpha} + q e^{i\beta} + p q e^{i(\alpha+\beta)} & p e^{i\alpha} + q e^{i\beta} + p q e^{i(\alpha+\beta)} & 1 + p^2 e^{2i\alpha} + q^2 e^{2i\beta} \end{pmatrix} \quad (2.6)$$

where we have parametrized the different elements of m_ν in terms of two real parameters p, q and two phase parameters α, β accordingly as

$$m_0 = -\frac{y_3^2}{m}, \quad p e^{i\alpha} = \frac{y_1}{y_3}, \quad q e^{i\beta} = \frac{y_2}{y_3}. \quad (2.7)$$

Upon diagonalization, m_ν yields degenerate mass eigenvalues [19]. The eigenvectors corresponding to the degenerate eigenvalues can not be determined uniquely. Hence the Diagonalization matrix is not unique and there by generates an ambiguity² in the neutrino mixing angles. Thus, it is necessary to break the discrete symmetry in order to accommodate neutrino oscillation data. Retaining the flavor diagonal texture of M_R , we introduce single nonzero symmetry breaking parameters ϵ in any of the diagonal entries. This can be done in three ways as

$$\begin{aligned} (i) \quad M_R &= \text{diag}(m, m, m(1 + \epsilon)) \\ (ii) \quad M_R &= \text{diag}(m, m(1 + \epsilon), m) \\ (iii) \quad M_R &= \text{diag}(m(1 + \epsilon), m, m) \end{aligned} \quad (2.8)$$

The m_ν matrices for the above mentioned three cases of symmetry breaking are (using the unique parametrization shown in eq.(2.7)) given by

²We put a brief explanation of this ambiguity in Ref. [19].

for case(i)

$$m_\nu = m_0 \begin{pmatrix} e^{2i\alpha}p^2 + e^{2i\beta}q^2 + \frac{1}{1+\epsilon} & e^{i\alpha}p + e^{i(\alpha+\beta)}pq + \frac{e^{i\beta}q}{1+\epsilon} & e^{i\beta}q + e^{i(\alpha+\beta)}pq + \frac{e^{i\alpha}p}{1+\epsilon} \\ e^{i\alpha}p + e^{i(\alpha+\beta)}pq + \frac{e^{i\beta}q}{1+\epsilon} & 1 + e^{2i\alpha}p^2 + \frac{e^{2i\beta}q^2}{1+\epsilon} & e^{i\alpha}p + e^{i\beta}q + \frac{e^{i(\alpha+\beta)}pq}{1+\epsilon} \\ e^{i\beta}q + e^{i(\alpha+\beta)}pq + \frac{e^{i\alpha}p}{1+\epsilon} & e^{i\alpha}p + e^{i\beta}q + \frac{e^{i(\alpha+\beta)}pq}{1+\epsilon} & 1 + e^{2i\beta}q^2 + \frac{e^{2i\alpha}p^2}{1+\epsilon} \end{pmatrix} \quad (2.9)$$

for case(ii)

$$m_\nu = m_0 \begin{pmatrix} 1 + e^{2i\alpha}p^2 + \frac{e^{2i\beta}q^2}{1+\epsilon} & e^{i\alpha}p + e^{i\beta}q + \frac{e^{i(\alpha+\beta)}pq}{1+\epsilon} & e^{i\alpha}p + e^{i(\alpha+\beta)}pq + \frac{e^{i\beta}q}{1+\epsilon} \\ e^{i\alpha}p + e^{i\beta}q + \frac{e^{i(\alpha+\beta)}pq}{1+\epsilon} & 1 + e^{2i\beta}q^2 + \frac{e^{2i\alpha}p^2}{1+\epsilon} & e^{i\beta}q + e^{i(\alpha+\beta)}pq + \frac{e^{i\alpha}p}{1+\epsilon} \\ e^{i\alpha}p + e^{i(\alpha+\beta)}pq + \frac{e^{i\beta}q}{1+\epsilon} & e^{i\beta}q + e^{i(\alpha+\beta)}pq + \frac{e^{i\alpha}p}{1+\epsilon} & e^{2i\alpha}p^2 + e^{2i\beta}q^2 + \frac{1}{1+\epsilon} \end{pmatrix} \quad (2.10)$$

for case(iii)

$$m_\nu = m_0 \begin{pmatrix} 1 + e^{2i\beta}q^2 + \frac{e^{2i\alpha}p^2}{1+\epsilon} & e^{i\beta}q + e^{i(\alpha+\beta)}pq + \frac{e^{i\alpha}p}{1+\epsilon} & e^{i\alpha}p + e^{i\beta}q + \frac{e^{i(\alpha+\beta)}pq}{1+\epsilon} \\ e^{i\beta}q + e^{i(\alpha+\beta)}pq + \frac{e^{i\alpha}p}{1+\epsilon} & e^{2i\alpha}p^2 + e^{2i\beta}q^2 + \frac{1}{1+\epsilon} & e^{i\alpha}p + e^{i(\alpha+\beta)}pq + \frac{e^{i\beta}q}{1+\epsilon} \\ e^{i\alpha}p + e^{i\beta}q + \frac{e^{i(\alpha+\beta)}pq}{1+\epsilon} & e^{i\alpha}p + e^{i(\alpha+\beta)}pq + \frac{e^{i\beta}q}{1+\epsilon} & 1 + e^{2i\alpha}p^2 + \frac{e^{2i\beta}q^2}{1+\epsilon} \end{pmatrix} \quad (2.11)$$

All the experimentally measurable observable (mass squared differences and mixing angles) of this broken symmetric neutrino mass matrix are obtained in terms of the Lagrangian parameters (p , q , α , β , m_0) and breaking parameter(ϵ) using the methodology developed in Ref. [19] to calculate the masses and mixing angles from the most general Majorana neutrino mass matrix.

3 Baryogenesis through leptogenesis

Here we will discuss about the lepton asymmetry arising from a CP asymmetry [24] generated due to the decay of heavy right handed Majorana neutrinos. At a high energy scale where $SU(2)_L \times U(1)_Y$ symmetry is not broken, physical right handed neutrinos N_{iR} with definite mass can decay into (i) charged lepton with charged scalar and (ii) light neutrino with neutral scalar. The conjugate decay process is also possible due to self conjugate nature of N_{iR} . A net lepton asymmetry will be generated if two decay processes occur at different rate. In the present case the right handed Majorana neutrinos are not hierarchical. Before the explicit breaking of the cyclic symmetry the mass spectrum of the right handed neutrinos is degenerate. After cyclic symmetry breaking the masses of the heavy right handed neutrinos differ by the symmetry breaking parameter(ϵ). Primarily we have varied the symmetry breaking parameter (ϵ) within a large range in order to fit neutrino oscillation data. With the motivation of keeping the symmetry breaking soft as well as to study the leptogenesis behaviour for quasi degenerate right handed neutrinos we pick only the smallest

value of ϵ (~ 0.004) allowed by the oscillation data. In this case the right handed neutrinos are nearly degenerate and there is a possibility of occurrence of resonant leptogenesis³ in this situation. This is the less addressed interesting case of leptogenesis, which we study in the present work. For higher values of ϵ (~ 0.1) right handed neutrinos are hierarchical and leptogenesis phenomena in this case has been well studied in the literature. In this work we explore the parameter region where the leptogenesis takes place due to decay of quasi degenerate right handed neutrinos. Unlike the strongly hierarchical case here we have to consider contributions from all three generations of right handed neutrinos [25] to calculate the CP asymmetry parameters.

3.1 Calculation of CP asymmetry parameters

The resummed effective Yukawa couplings (considering three generations of N_{iR}) are given by [25, 26]

$$(\bar{h}_+^\nu)_{\alpha i} = h_{\alpha i}^\nu + iB_{\alpha i} - i \sum_{j,k=1}^3 |\epsilon_{ijk}| h_{\alpha j}^\nu \times \frac{m_{N_i}(m_{N_i}A_{ij} + m_{N_j}A_{ji}) + R_{ik}[m_{N_i}A_{kj}(m_{N_i}A_{ik} + m_{N_k}A_{ki}) + m_{N_j}A_{jk}(m_{N_i}A_{ki} + m_{N_k}A_{ik})]}{m_{N_i}^2 - m_{N_j}^2 + 2i m_{N_i}^2 A_{jj} + 2i \text{Im}R_{ik} (m_{N_i}^2 |A_{jk}|^2 + m_{N_j} m_{N_k} \text{Re}A_{jk}^2)}, \quad (3.1)$$

where

$$R_{ij} = \frac{m_{N_i}^2}{m_{N_i}^2 - m_{N_j}^2 + 2i m_{N_i}^2 A_{jj}}, \quad (3.2)$$

$$A_{ij} = \frac{(h^{\nu\dagger} h^\nu)_{ji}}{16\pi}, \quad (3.3)$$

$$B_{\alpha i} = - \sum_{(j \neq i)} \frac{(h^{\nu\dagger} h^\nu)_{ij} h_{\alpha j}^\nu}{16\pi} f\left(\frac{m_{N_j}^2}{p^2}\right), \quad (3.4)$$

$(h^\nu)_{\alpha i}$ is tree level neutrino Yukawa coupling and $|\epsilon_{ijk}|$ is the modulus of the usual Levi-Civita anti-symmetric tensor.

The resummed effective amplitudes for the decays $N_{iR} \rightarrow l_\alpha \Phi$ are denoted as $\mathcal{T}(N_{iR} \rightarrow l_\alpha \Phi)$ and are given by

$$\mathcal{T}(N_{iR} \rightarrow l_\alpha \Phi) = (\bar{h}_+^\nu)_{\alpha i} \bar{u}_\alpha P_R u_{N_i}, \quad (3.5)$$

³Only the smallness of the breaking parameter doesn't guarantee the resonance enhancement of CP asymmetry, resonance occurs only when the resonance condition is satisfied.

where i ($i = 1, 2, 3$) and α ($\alpha = e, \mu, \tau$) are the generation indices of N_{iR} and leptons respectively and u_α, u_{N_i} denote corresponding spinorial fields. The CP conjugate decay amplitudes $\mathcal{T}(N_{iR} \rightarrow l_\alpha^c \Phi^\dagger)$ can be obtained easily from eq.(3.5) by replacing $(\bar{h}_+^\nu)_{\alpha i}$ with $(\bar{h}_-^\nu)_{\alpha i}$ which can be further recovered from eq.(3.1) by taking complex conjugate of the Yukawa couplings. The CP asymmetry of the decay is characterized by a parameter ε_i^α defined by

$$\begin{aligned}\varepsilon_i^\alpha &= \frac{\Gamma(N_{iR} \rightarrow l_\alpha \Phi) - \Gamma(N_{iR} \rightarrow l_\alpha^c \Phi^\dagger)}{\Sigma_\alpha [\Gamma(N_{iR} \rightarrow l_\alpha \Phi) + \Gamma(N_{iR} \rightarrow l_\alpha^c \Phi^\dagger)]} \\ &= \frac{(\bar{h}_+^{\nu\dagger})_{i\alpha} (\bar{h}_+^\nu)_{\alpha i} - (\bar{h}_-^{\nu\dagger})_{i\alpha} (\bar{h}_-^\nu)_{\alpha i}}{(\bar{h}_+^{\nu\dagger} \bar{h}_+^\nu)_{ii} + (\bar{h}_-^{\nu\dagger} \bar{h}_-^\nu)_{ii}}.\end{aligned}\quad (3.6)$$

After a long algebraic manipulation the expression of ε_i^α is presented in a simpler form keeping terms upto $O(h^{\nu 4})$ as

$$\begin{aligned}\varepsilon_i^\alpha &= \frac{1}{4\pi v^2 H_{ii}} \sum_{j \neq i} \text{Im}\{H_{ij}(m_D^\dagger)_{i\alpha}(m_D)_{\alpha j}\} \left[f(x_{ij}) + \frac{\sqrt{x_{ij}}(1-x_{ij})}{(1-x_{ij})^2 + \frac{H_{jj}^2}{16\pi^2 v^4}} \right] \\ &+ \frac{1}{4\pi v^2 H_{ii}} \sum_{j \neq i} \frac{(1-x_{ij})\text{Im}\{H_{ji}(m_D^\dagger)_{i\alpha}(m_D)_{\alpha j}\}}{(1-x_{ij})^2 + \frac{H_{jj}^2}{16\pi^2 v^4}}\end{aligned}\quad (3.7)$$

where $m_D = \frac{vh^\nu}{\sqrt{2}}$, $H = (m_D^\dagger m_D)$, $x_{ij} = \frac{m_{N_j}^2}{m_{N_i}^2}$ and $f(x_{ij})$ is the loop function given by

$$f(x_{ij}) = \sqrt{x_{ij}} \left\{ 1 - (1+x_{ij}) \ln\left(\frac{1+x_{ij}}{x_{ij}}\right) \right\}.\quad (3.8)$$

Again, derived expressions for ε_i^α are quite general and can be used for hierarchical as well as quasi degenerate mass spectrum (without or with resonant conditions like $1 - x_{ij} \simeq \frac{H_{jj}}{4\pi v^2}$) of right handed neutrinos. For hierarchical case one can simplify ε_i^α to standard formula [27] neglecting $\frac{H_{jj}^2}{16\pi^2 v^4}$ compared to $(1-x_{ij})^2$. We have neglected $O(h^{\nu 6})$ and higher order terms in our obtained expression of ε_i^α in eq.(3.7). Contribution of those terms are negligible for most of the cases.

3.2 Boltzmann equations for leptogenesis

In the present work the right handed neutrinos are taken to be nearly degenerate. Thus in a temperature regime where the lepton flavors are distinguishable, we have to consider the flavor dependent as well as resonant leptogenesis formalism. The corresponding set of Boltzmann equations are given by [25]

$$\begin{aligned}\frac{d\eta_{N_i}}{dz} &= \frac{z}{H(z=1)} \left[\left(1 - \frac{\eta_{N_i}}{\eta_{N_i}^{\text{eq}}} \right) \sum_\alpha \left(\Gamma^{\alpha D(i)} + \Gamma_{\text{Yukawa}}^{\alpha S(i)} + \Gamma_{\text{Gauge}}^{\alpha S(i)} \right) \right. \\ &\quad \left. - \frac{1}{4} \sum_\alpha \eta_L^\alpha \varepsilon_i^\alpha \left(\Gamma^{\alpha D(i)} + \tilde{\Gamma}_{\text{Yukawa}}^{\alpha S(i)} + \tilde{\Gamma}_{\text{Gauge}}^{\alpha S(i)} \right) \right],\end{aligned}\quad (3.9)$$

$$\begin{aligned} \frac{d\eta_L^\alpha}{dz} = & -\frac{z}{H(z=1)} \left\{ \sum_{i=1}^3 \varepsilon_i^\alpha \left(1 - \frac{\eta_{N_i}}{\eta_{N_i}^{\text{eq}}} \right) \sum_{\beta} \left(\Gamma^{\beta D(i)} + \Gamma_{\text{Yukawa}}^{\beta S(i)} + \Gamma_{\text{Gauge}}^{\beta S(i)} \right) \right. \\ & \left. + \frac{1}{4} \eta_L^\alpha \left[\sum_{i=1}^3 \left(\Gamma^{\alpha D(i)} + \Gamma_{\text{Yukawa}}^{\alpha W(i)} + \Gamma_{\text{Gauge}}^{\alpha W(i)} \right) + \Gamma_{\text{Yukawa}}^{\alpha \Delta L=2} \right] \right\}, \end{aligned} \quad (3.10)$$

where

$z = \frac{\text{mass of lightest right handed neutrino}}{\text{temperature}} = \frac{m_{N_1}}{T}$ and the parameter η_a give the number density of a particle species a normalized to the photon density, i.e $\eta_a(z) = \frac{n_a(z)}{n_\gamma(z)}$ and $\eta_a^{\text{eq}}(z) = \frac{n_a^{\text{eq}}(z)}{n_\gamma(z)}$ with $n_\gamma(z) = \frac{2m_{N_1}^3}{\pi^2 z^3}$. The number density of a particle species a with g_a internal degrees of freedom is given by

$$n_a(T) = \frac{g_a m_a^2 T e^{\mu_a(T)/T}}{2\pi^2} K_2\left(\frac{m_a}{T}\right) \quad (3.11)$$

and $n_a(T)$ satisfies the equilibrium density condition when $\mu_a = 0$, i.e [28]

$$n_a^{\text{eq}}(T) = \frac{g_a m_a^2 T}{2\pi^2} K_2\left(\frac{m_a}{T}\right). \quad (3.12)$$

Here a denotes a definite particle species. The various Γ s in the RHS of the Boltzmann equations are normalized (by photon density) decays and scattering cross sections [25],

$$\begin{aligned} \Gamma^{\alpha D(i)} &= \frac{1}{n_\gamma} \gamma_{L^\alpha \Phi}^{N_i}, \\ \Gamma_{\text{Yukawa}}^{\alpha S(i)} &= \frac{1}{n_\gamma} \left(\gamma_{Qu^C}^{N_i L^\alpha} + \gamma_{L^\alpha Q^C}^{N_i u} + \gamma_{L^\alpha u}^{N_i Q} \right), \\ \tilde{\Gamma}_{\text{Yukawa}}^{\alpha S(i)} &= \frac{1}{n_\gamma} \left(\frac{\eta_{N_i}}{\eta_{N_i}^{\text{eq}}} \gamma_{Qu^C}^{N_i L^\alpha} + \gamma_{L^\alpha Q^C}^{N_i u} + \gamma_{L^\alpha u}^{N_i Q} \right), \\ \Gamma_{\text{Gauge}}^{\alpha S(i)} &= \frac{1}{n_\gamma} \left(\gamma_{L^\alpha \Phi}^{N_i V_\mu} + \gamma_{\Phi^\dagger V_\mu}^{N_i L^\alpha} + \gamma_{L^\alpha V_\mu}^{N_i \Phi^\dagger} \right), \\ \tilde{\Gamma}_{\text{Gauge}}^{\alpha S(i)} &= \frac{1}{n_\gamma} \left(\gamma_{L^\alpha \Phi}^{N_i V_\mu} + \frac{\eta_{N_i}}{\eta_{N_i}^{\text{eq}}} \gamma_{\Phi^\dagger V_\mu}^{N_i L^\alpha} + \gamma_{L^\alpha V_\mu}^{N_i \Phi^\dagger} \right), \\ \Gamma_{\text{Yukawa}}^{\alpha W(i)} &= \frac{2}{n_\gamma} \left(\gamma_{Qu^C}^{N_i L^\alpha} + \gamma_{L^\alpha Q^C}^{N_i u} + \gamma_{L^\alpha u}^{N_i Q} + \frac{\eta_{N_i}}{2\eta_{N_i}^{\text{eq}}} \gamma_{Qu^C}^{N_i L^\alpha} \right), \\ \Gamma_{\text{Gauge}}^{\alpha W(i)} &= \frac{2}{n_\gamma} \left(\gamma_{L^\alpha \Phi}^{N_i V_\mu} + \gamma_{\Phi^\dagger V_\mu}^{N_i L^\alpha} + \gamma_{L^\alpha V_\mu}^{N_i \Phi^\dagger} + \frac{\eta_{N_i}}{2\eta_{N_i}^{\text{eq}}} \gamma_{\Phi^\dagger V_\mu}^{N_i L^\alpha} \right), \\ \Gamma_{\text{Yukawa}}^{\alpha \Delta L=2} &= \frac{2}{n_\gamma} \sum_{\beta} \left(\gamma_{L^{\beta C} \Phi^\dagger}^{N_i L^\alpha \Phi} + 2\gamma_{\Phi^\dagger \Phi^\dagger}^{L^\alpha L^\beta} \right) \end{aligned} \quad (3.13)$$

where α denotes lepton flavor indices (e, μ, τ). For a generic process $X \rightarrow Y$, γ_Y^X is defined as

$$\gamma_Y^X \equiv \gamma(X \rightarrow Y) + \gamma(\bar{X} \rightarrow \bar{Y}), \quad (3.14)$$

with

$$\gamma(X \rightarrow Y) = \int d\pi_X d\pi_Y (2\pi)^4 \delta^{(4)}(p_X - p_Y) e^{-p_X^0/T} |\mathcal{M}(X \rightarrow Y)|^2. \quad (3.15)$$

The explicit expressions of different γ s listed above are taken from the appendix of [25] ⁴.

When the resonance condition ($1 - x_{ij} = \frac{H_{jj}}{4\pi v^2}$) is not satisfied strictly, such that the enhancement of the CP asymmetry parameter (ε_i^α) is not too high, contribution of the second term of eq.(3.9) is negligible compared to the first term. In that case the first Boltzmann equation (3.9) can be rewritten in a simpler form as

$$\frac{d\eta_{N_i}(z)}{dz} = (D_i(z) + D_i^{\text{SY}}(z) + D_i^{\text{SG}}(z))(\eta_{N_i}^{\text{eq}}(z) - \eta_{N_i}(z)). \quad (3.16)$$

In terms of another parameter Y_{N_i} ($= n_{N_i}/s$, where n_{N_i} is the number density of N_i and s is the comoving entropy density) ⁵ the above equation can be rewritten as

$$\frac{dY_{N_i}(z)}{dz} = (D_i(z) + D_i^{\text{SY}}(z) + D_i^{\text{SG}}(z))(Y_{N_i}^{\text{eq}}(z) - Y_{N_i}(z)) \quad (3.17)$$

where

$$\begin{aligned} D_i(z) &= \sum_{\alpha} D_i^{\alpha}(z) = \sum_{\alpha} \frac{z}{H(z=1)} \frac{\Gamma^{\alpha D(i)}}{\eta_{N_i}^{\text{eq}}(z)}, \\ D_i^{\text{SY}}(z) &= \sum_{\alpha} \frac{z}{H(z=1)} \frac{\Gamma_{\text{Yukawa}}^{\alpha S(i)}}{\eta_{N_i}^{\text{eq}}(z)}, \\ D_i^{\text{SG}}(z) &= \sum_{\alpha} \frac{z}{H(z=1)} \frac{\Gamma_{\text{Gauge}}^{\alpha S(i)}}{\eta_{N_i}^{\text{eq}}(z)}. \end{aligned}$$

Similarly (neglecting $\Delta L = 2$ scattering terms) the second Boltzmann equation (3.10) can be written as

$$\begin{aligned} \frac{d\eta_L^{\alpha}(z)}{dz} = & - \left\{ \sum_{i=1}^3 \varepsilon_i^{\alpha} (D_i(z) + D_i^{\text{SY}}(z) + D_i^{\text{SG}}(z))(\eta_{N_i}^{\text{eq}}(z) - \eta_{N_i}(z)) \right. \\ & + \left. \frac{1}{4} \eta_L^{\alpha} \sum_{i=1}^3 \left(\frac{1}{2} D_i^{\alpha}(z) z^2 K_2(z) + D_i^{\alpha \text{YW}}(z) + D_i^{\alpha \text{GW}}(z) \right) \right\} \end{aligned} \quad (3.18)$$

with

$$\begin{aligned} D_i^{\text{YW}}(z) &= \sum_{\alpha} \frac{z}{H(z=1)} \Gamma_{\text{Yukawa}}^{\alpha W(i)}, \\ D_i^{\text{GW}}(z) &= \sum_{\alpha} \frac{z}{H(z=1)} \Gamma_{\text{Gauge}}^{\alpha W(i)}. \end{aligned}$$

⁴Those expressions are free of flavor index α . We have introduced the flavor indices in suitable places.

⁵ $\eta_{N_i}(z) = 1.8g_{*s}(T)Y_{N_i}(z)$, but in our regime of interest $g_{*s}(T)$ is nearly constant and thus we can say that $\eta_{N_i}(z)$ and $Y_{N_i}(z)$ are connected through a constant factor.

The second Boltzmann equation governs the evolution of the lepton flavor asymmetry (η_L^α). Now we will discuss the recipe to calculate the baryon asymmetry from lepton flavor asymmetry. At first we introduce a parameter Y_α which is number density (of a lepton flavor) normalized by entropy density (s) and it is related to η_L^α through

$$Y_\alpha = \frac{n_L^\alpha - n_{\bar{L}}^\alpha}{s} = \left(\frac{\eta_\gamma}{s}\right)\eta_L^\alpha. \quad (3.19)$$

We know that $\frac{s}{\eta_\gamma} = 1.8g_{*s}$ [29] where g_{*s} counts total number of effective massless degrees of freedom and it is a function of temperature. For $T > 10^2$ GeV, g_{*s} is nearly constant and its value (with three right handed neutrinos) is 112 [30]. The lepton asymmetry created by the decay of right handed neutrinos is converted into baryon asymmetry through sphaleron process. During the sphaleron process the quantity $\Delta_\alpha = \frac{B}{3} - L^\alpha$ (where B is the baryon number and L is the lepton number) is conserved. The Y_{Δ_α} asymmetries and Y_α asymmetries are related through a matrix ‘ A ’ as $Y_\alpha = \sum_\beta A_{\alpha\beta} Y_{\Delta_\beta}$. The Boltzmann equation (3.18) governing the evolution of flavor asymmetries can be written in terms of Y_{Δ_α} parameters as

$$\begin{aligned} \frac{dY_{\Delta_\alpha}}{dz} = & \sum_{i=1}^3 \{ \varepsilon_i^\alpha (D_i(z) + D_i^{\text{SY}}(z) + D_i^{\text{SG}}(z))(Y_{N_i}^{\text{eq}}(z) - Y_{N_i}(z)) \} \\ & + \frac{\sum_\beta A_{\alpha\beta} Y_{\Delta_\beta}}{4} \left\{ \sum_{i=1}^3 \left(\frac{1}{2} D_i^\alpha(z) z^2 K_2(z) + D_i^{\alpha \text{YW}}(z) + D_i^{\alpha \text{GW}}(z) \right) \right\}. \end{aligned} \quad (3.20)$$

We now solve the set of coupled Boltzmann equations (given in eq.(3.17) and eq.(3.20)) upto a value of z where the quantities Y_{Δ_α} attain a constant value.

4 Calculation of baryon asymmetry in different energy regimes

We are now in a position to compute the baryon asymmetry in different regimes [31, 32].

4.0.1 $M_{\text{lowest}} < 10^9$ GeV.

Here all three lepton flavors are separately active. Thus the A matrix connecting Y_{Δ_α} and Y_α is a 3×3 matrix given by [31]

$$A = \begin{pmatrix} -151/179 & 20/179 & 20/179 \\ 25/358 & -344/537 & 14/537 \\ 25/358 & 14/537 & -344/537 \end{pmatrix}. \quad (4.1)$$

The final baryon asymmetry Y_B (baryon asymmetry normalized by entropy density) is given by [33]

$$Y_B = \frac{28}{79} \sum_\alpha Y_{\Delta_\alpha}. \quad (4.2)$$

Another important parameter, i.e baryon asymmetry normalized to photon density is obtained as

$$\eta_B = \left. \frac{s}{n_\gamma} \right|_0 Y_B = 7.0394 Y_B, \quad (4.3)$$

where $\eta_B = \left. \frac{s}{n_\gamma} \right|_0 Y_B = 7.0394 Y_B$, the zero at the subscript denotes its value at present epoch.

4.0.2 $10^9 \text{ GeV} < M_{\text{lowest}} < 10^{12} \text{ GeV}$.

In this regime τ flavor is distinguishable but we can't differentiate between e and μ flavors. So we define two CP asymmetries $\varepsilon_{N_i}^\tau$ and $\varepsilon_{N_i}^2 = \varepsilon_{N_i}^e + \varepsilon_{N_i}^\mu$ and the corresponding lepton flavor asymmetry parameters are Y_τ and $Y_2 = Y_e + Y_\mu$. Solving the Boltzmann equations (3.17,3.20) we get two Y_Δ asymmetries (Y_{Δ_2} and Y_{Δ_τ}) and the final baryon asymmetry parameter is calculated as

$$Y_B = \frac{28}{79} (Y_{\Delta_2} + Y_{\Delta_\tau}). \quad (4.4)$$

In this case the A matrix connecting Y_α and Y_{Δ_α} is a 2×2 matrix given by [31]

$$A = \begin{pmatrix} -417/589 & 120/589 \\ 30/589 & -390/589 \end{pmatrix}. \quad (4.5)$$

4.0.3 $M_{\text{lowest}} > 10^{12} \text{ GeV}$.

All the charged lepton flavors act indistinguishably in this regime and therefore one can define a single CP asymmetry parameter $\varepsilon_i = \sum_\alpha \varepsilon_i^\alpha$. The other α dependent terms in RHS of eq.(3.20) are replaced by sum over α and the A matrix is taken as negative unit matrix. So the Boltzmann equation (3.20) is now free of the flavor index α and solving the same we get a single Y_Δ . The final baryon asymmetry parameter is obtained as

$$Y_B = \frac{28}{79} Y_\Delta. \quad (4.6)$$

5 Numerical results and phenomenological discussion

For numerical estimation of baryon asymmetry we need to know CP asymmetry parameters ε_i^α and various decays and scattering cross sections in terms of the parameters m , m_0 , p , q , α , β and ϵ . However, dependencies of those parameters on ε_i^α and the decay/scattering terms arise through the expressions of m_D , $H(=m_D^\dagger m_D)$ and x_{ij} . Obviously it is then necessary to express, m_D , H and x_{ij} in terms of those Lagrangian parameters. Utilizing eq.(2.7) we explicitly express the elements of m_D in terms of the aforesaid parameters through

$$y_3 = i\sqrt{mm_0}, \quad y_1 = i\sqrt{mm_0}pe^{i\alpha}, \quad \text{and} \quad y_2 = i\sqrt{mm_0}qe^{i\beta} \quad (5.1)$$

which in effect gives

$$\begin{aligned}
m_D &= i\sqrt{mm_0} \begin{pmatrix} pe^{i\alpha} & qe^{i\beta} & 1 \\ 1 & pe^{i\beta} & qe^{i\beta} \\ qe^{i\beta} & 1 & pe^{i\alpha} \end{pmatrix}, \\
H &= m_D^\dagger m_D \\
&= \begin{pmatrix} |y_1|^2 + |y_2|^2 + |y_3|^2 & y_1^* y_2 + y_1 y_3^* + y_2^* y_3 & y_1 y_2^* + y_1^* y_3 + y_2 y_3^* \\ y_1 y_2^* + y_1^* y_3 + y_2 y_3^* & |y_1|^2 + |y_2|^2 + |y_3|^2 & y_1^* y_2 + y_1 y_3^* + y_2^* y_3 \\ y_1^* y_2 + y_1 y_3^* + y_2^* y_3 & y_1 y_2^* + y_1^* y_3 + y_2 y_3^* & |y_1|^2 + |y_2|^2 + |y_3|^2 \end{pmatrix} \\
&= mm_0 \begin{pmatrix} X & Y & Y^* \\ Y^* & X & Y \\ Y & Y^* & X \end{pmatrix}
\end{aligned} \tag{5.2}$$

with

$$\begin{aligned}
X &= 1 + p^2 + q^2 \\
Y &= pe^{i\alpha} + qe^{-i\beta} + pqe^{i(\beta-\alpha)}.
\end{aligned} \tag{5.3}$$

Again $x_{ij} = m_{N_j}^2/m_{N_i}^2$ (and $x_{ji} = 1/x_{ij}$) is estimated from M_R as

$$\begin{aligned}
x_{12} &= 1, \quad x_{23} = x_{13} = (1 + \epsilon)^2 \quad \text{for case(i)} \\
x_{12} &= \frac{1}{x_{23}} = (1 + \epsilon)^2, \quad x_{13} = 1 \quad \text{for case(ii)} \\
x_{12} &= x_{13} = \frac{1}{(1 + \epsilon)^2}, \quad x_{23} = 1 \quad \text{for case(iii)}.
\end{aligned} \tag{5.4}$$

To find out the allowed parameter space we adopt the following methodology. In the present work the parameter space is constrained due to the bound on the frozen value Baryon asymmetry parameter (η_B or Y_B) keeping in mind all the neutrino oscillation experimental data (Table 1). The parameter space is constrained in two steps. At first all the neutrino physics observables (mass eigenvalues, mixing angles) are expressed in terms of the Lagrangian parameters (m_0, p, q, α, β) and the breaking parameter (ϵ)⁶. In the first step the parameters get restricted by the experimental ranges of neutrino mass squared differences (solar and atmospheric) and mixing angles. These constrained set of parameters are used thereafter to calculate the CP asymmetry parameters and hence the baryon asymmetry parameter Y_B (or η_B) for different values of right handed neutrino mass m (to take into account fully flavored, τ -flavored and unflavored leptogenesis). Hence, the parameters get second round of restriction from the limits on baryon asymmetry. We have observed

⁶Our parameter space satisfy i) the cosmological bound on sum mass $\sum m_i (= m_1 + m_2 + m_3) < (0.23 - 1.11)eV$ [34] with PLANCK [9] and other cosmological observations [35], [36] ii) also the bound $|m_{\nu_{ee}}| < (0.14 - 0.38)eV$ [37] of neutrinoless double beta decay experiments [18, 38, 39].

Table 1: Input data from neutrino oscillation experiments [18]

Quantity	3σ ranges/other constraint
Δm_{21}^2	$7.12 < \Delta m_{21}^2 (10^5 \text{ eV}^{-2}) < 8.20$
$ \Delta m_{31}^2 (N)$	$2.31 < \Delta m_{31}^2 (10^3 \text{ eV}^{-2}) < 2.74$
$ \Delta m_{31}^2 (I)$	$2.21 < \Delta m_{31}^2 (10^3 \text{ eV}^{-2}) < 2.64$
θ_{12}	$31.30^\circ < \theta_{12} < 37.46^\circ$
θ_{23}	$36.86^\circ < \theta_{23} < 55.55^\circ$
θ_{13}	$7.49^\circ < \theta_{13} < 10.46^\circ$

that apart from the two parameter breaking of cyclic symmetry($\epsilon_1, \epsilon_2 \neq 0$) (which is studied extensively in Ref [19]) one parameter breaking is also well fitted by the extant data. In this work our main motivation is to study the resonance enhancement of the CP asymmetries. So we have tried to keep the masses of the right handed neutrinos as close as allowed by the oscillation data. In order to pin down the parameter space for each type of leptogenesis we consider three categories of single parameter cyclic symmetry breaking designated by case(i), (ii) and (iii) in eq.(2.8).

Before going into the case wise details it is worthwhile to mention that

1. We have studied the variation of CP asymmetry parameters (ϵ_i^α) with right handed neutrino mass (m) in all the cases of symmetry breaking mentioned above. It is found that $|\epsilon_i^\alpha|$ vs m curve shows a resonance peak near $m = 10^{12}$ GeV. Resonance is achieved when the condition $(1 - x_{ij}) = \frac{H_{jj}}{4\pi v^2}$ is satisfied which gives $m_{res} \simeq \frac{8\pi v^2 \epsilon}{m_0}$ at the point of resonance.
2. When the parameter space is constrained with the neutrino oscillation data it is seen that the value of m_0 decreases as the value (no matter positive/negative) of breaking parameter (ϵ) is increased. Now the condition for resonance enhancement of CP asymmetry is given by $m_{res} \simeq \frac{8\pi v^2 \epsilon}{m_0}$. Therefore for a larger value of ϵ the mass of right handed neutrino required for resonance will also be bigger. The value of m_{res} is $\sim 10^{12}$ GeV for the lowest allowed value of $\epsilon(= -0.004)$ ⁷. So for any value of $|\epsilon| > 0.004$, m_{res} will also be greater than 10^{12} GeV which falls in the unflavored regime of leptogenesis and our breaking scheme and the imposed symmetry is such that it will produce a null asymmetry in this regime. Therefore even if we take a bigger value of ϵ we can't observe the effect of resonance since the corresponding m_{res} is in the unflavored regime. To get an m_{res} in the flavored leptogenesis regime we have to take $|\epsilon|$ smaller than 0.004 which is again not allowed by the oscillation data. So we prefer to

⁷In the cases we have analyzed in section 5 the resonance enhancement of CP asymmetry parameter takes place near $m \sim 10^{12}$ GeV, but we cannot see its effect in producing baryon asymmetry since it is in the unflavored regime.

carry out the analysis in a region where the symmetry is softly broken.

It is worthwhile to make a small remark in this context. The maximum value of breaking parameter allowed by the oscillation data (for case(i) of symmetry breaking) is $\epsilon = -0.78$ and for it $m_0 \sim 3.5 \times 10^{-4}$ eV. The right handed neutrino mass required for resonance comes out to be $m_{res} \sim 3.38 \times 10^{18}$ GeV which is beyond the scope of fully flavored and τ -flavored leptogenesis.

3. To get an upper bound on right handed neutrino mass we use the perturbative unitarity limit [40], i.e

$$\begin{aligned} \frac{(h_{ij}^\nu)^2}{4\pi} &< 1 \\ \Rightarrow (m_D)_{ij}^2 &< 2\pi v^2. \end{aligned} \quad (5.5)$$

According to our parametrization

$$m = -\frac{y_3^2}{m_0}. \quad (5.6)$$

Since y_3 is an element of m_D matrix, we can write

$$|m| < \frac{2\pi v^2}{m_0} \quad (5.7)$$

In our cases of interest this condition sets the upper limit of right handed neutrino mass near 10^{14} GeV.

4. To solve Boltzmann equations we have considered two initial conditions $Y_{N_i} = 0$ and $Y_{\Delta_\alpha} = 0$. These mean that initially there were no lepton asymmetry and right handed neutrinos. Leptons at first produce appreciable amount of right handed neutrinos which decay asymmetrically to leptons. Other conditions, frequently used in the literature, are $Y_{N_i} = Y_{N_i}^{eq}$ and $Y_{\Delta_\alpha} = 0$. But we restrict ourselves to the previous one to solve the equations.

5.1 Numerical analysis for case(i) of symmetry breaking

It is implemented by incorporating the symmetry breaking parameter ϵ in the ‘33’ element of M_R as shown in case(i) of eq.(2.8). The calculation of mixing angles and mass eigenvalues using the resulting mass matrix (2.9) is carried out thereafter. The parameter space, constrained by the extant data, is used to find out the numerical value of the baryon asymmetry Y_B . By varying the mass of the right chiral neutrinos, we have studied leptogenesis in all three energy regimes as mentioned earlier.

As mentioned earlier, we are interested in the study of enhancement of the CP asymmetry parameter for the nearly degenerate right handed neutrinos we pick only those set of Lagrangian parameters $\{p, q, \alpha, \beta, m_0\}$ corresponding to the lowest allowed (by oscillation data) modulus value of the breaking parameter ϵ . We proceed further to calculate the Y_B for those restricted sets of values only. After the first round of restriction (by oscillation data) it is found that the lowest allowed value of the breaking parameter is $\epsilon = -0.004$ and for this value of ϵ we get 10 sets of values of the parameters $\{p, q, \alpha, \beta, m_0\}$. (There are two sets of values of $\{p, q, \alpha, \beta\}$ and for each set, there are five different values of m_0 . Therefore all total we have ten sets of values which are shown in Table 2.) For this case (i) normal hierarchy is preferred and θ_{23} is selected in the first octant (37.64°). But the sign of α, β remain unsettled. This in effect produces sign ambiguity in Dirac CP phase: $\delta_{CP} = 29.38^\circ$ for the set 1 with ($\alpha = -113.5^\circ, \beta = 120.5^\circ$) and $\delta_{CP} = -29.38^\circ$ for the set 2 with ($\alpha = 113.5^\circ, \beta = -120.5^\circ$).

To solve the sign ambiguity of the phases and to restrict the right handed neutrino mass scale m we carry out the required calculation for baryogenesis through leptogenesis in three different energy regimes using these set of 10 values only. We have one free parameter in hand, i.e the mass of lowest right handed neutrino which is chosen according to the energy regime we are working in. In the **fully flavored** case the mass of the lightest right handed neutrino is less than 10^9

Table 2: Sets of parameters allowed by oscillation data for case(i) of symmetry breaking with $\epsilon = -0.004$

sets	parameters					
	p	q	α (deg.)	β (deg.)	$m_0 \times 10^9$ (GeV)	$\frac{m}{10^7}$ (GeV)
1	0.97	0.89	-113.5	120.5	1.563	unrestricted for all m_0 values
					1.584	
					1.606	
					1.627	
					1.648	
2	0.97	0.89	113.5	-120.5	1.563	unrestricted for all m_0 values
					1.584	
					1.606	
					1.627	
					1.648	

Gev. All three lepton flavors (e, μ, τ) are separately active in this regime. Here we have to solve the set of flavor dependent coupled Boltzmann equations (3.17, 3.20) for three flavors (e, μ, τ)

separately to get the evolution of the flavor asymmetries (Y_{Δ_e} , Y_{Δ_μ} , Y_{Δ_τ}) with z . The RHS of the Boltzmann equations are known in terms of the Lagrangian parameters which are already restricted by oscillation data. After obtaining the Y_{Δ_α} asymmetry parameters we have to follow the steps given in subsection 4.0.1 to get the final value of the baryon asymmetry scaled by entropy density (Y_B)⁸. Evolution of $Y_B(= \frac{n_B - n_{\bar{B}}}{s})$ with z is computed with each set of values of the parameters $\{p, q, \alpha, \beta, m_0\}$ for different values of right handed neutrino mass m . It is found that while using the first five sets (i.e set 1 of $\{p, q, \alpha, \beta\}$ with five different m_0) Y_B attains a constant positive value at a high z whereas Y_B freezes at negative value when the calculation is done with next five sets (i.e set 2 of $\{p, q, \alpha, \beta\}$ with five different m_0). Therefore set 2 of $\{p, q, \alpha, \beta\}$ can readily be discarded since experimental observations have confirmed the fact that baryon asymmetry at present epoch must be positive. For the first five sets final value of Y_B is calculated (for different right handed neutrino masses) and tabulated below (Table 3). As an example we pick the case

Table 3: Final value of baryon asymmetry for set 1 of $\{p, q, \alpha, \beta\}$ (m and m_0 are in GeV)

$m_0 \times 10^9 \backslash \frac{m}{10^7}$	$Y_B \times 10^{11}$							
	2.0	2.2	2.4	2.5	2.6	2.7	2.9	3.0
1.563	7.00	7.70	8.41	8.76	9.10	9.46	10.16	10.51
1.584	7.11	7.83	8.54	8.89	9.25	9.60	10.31	10.67
1.606	7.22	7.94	8.67	9.03	9.39	9.75	10.47	10.83
1.627	7.33	8.06	8.79	9.16	9.53	9.89	10.63	10.99
1.648	7.43	8.18	8.92	9.29	9.66	10.03	10.78	11.15

($m = 2.4 \times 10^7$ GeV, $m_0 = 1.606 \times 10^{-9}$ GeV) (for which Y_B is within the experimental range) and show the variation of flavor asymmetries (Y_{Δ_α}) and baryon asymmetry (Y_B) with z in Fig.1. The sign of different asymmetries (Y_{Δ_e} , Y_{Δ_μ} , Y_{Δ_τ}) for various values of z are shown in the Table 4. An important observation of our numerical estimation should also be mentioned in this context that $|Y_{\Delta_\tau}|$ is always greater than $|Y_{\Delta_e} + Y_{\Delta_\mu}|$. It is clear from Table 4 that same quantities in set 1 and set 2 bears a relative opposite sign⁹. Here we are showing the plots for set 1 only, which survives the baryon asymmetry bound.

⁸Final value of Y_B means it is the frozen value of $Y_B = (n_B - n_{\bar{B}})/s$ which is its value at present epoch and it is related to frozen value of $\eta_B = (n_B - n_{\bar{B}})/n_\gamma$ as $\eta_B = 7.04 Y_B$.

⁹Here set 1 corresponds to $\{p, q, \alpha, \beta, m_0\}$ and set 2 corresponds to $\{p, q, -\alpha, -\beta, m_0\}$. Examining expression of the flavor dependent CP asymmetry parameter ε_i^α (eq.(3.7)) we find that apart from p, q, m_0 it contains *sine* functions of α, β and $(\alpha \pm \beta)$. Therefore for example if $\varepsilon_i^\alpha(p, q, \alpha, \beta, m_0)$ is positive, $\varepsilon_i^\alpha(p, q, -\alpha, -\beta, m_0)$ must be negative. These ε_i^α s are then inserted in the Boltzmann equation (eq.(3.20)) to find the flavor dependent Y_{Δ_α} asymmetries. This is why the relative opposite sign appears between the same quantities of set 1 and set 2.

Table 4: signs of different asymmetries at different z values

	$z = 0.01 \rightarrow 0.02$				$z = 0.02 \rightarrow 0.5$				$z > 0.5$			
	Y_{Δ_e}	Y_{Δ_μ}	Y_{Δ_τ}	Y_B	Y_{Δ_e}	Y_{Δ_μ}	Y_{Δ_τ}	Y_B	Y_{Δ_e}	Y_{Δ_μ}	Y_{Δ_τ}	Y_B
set 1	-ve	-ve	+ve	+ve	+ve	+ve	-ve	-ve	-ve	-ve	+ve	+ve
set 2	+ve	+ve	-ve	-ve	-ve	-ve	+ve	+ve	+ve	+ve	-ve	-ve

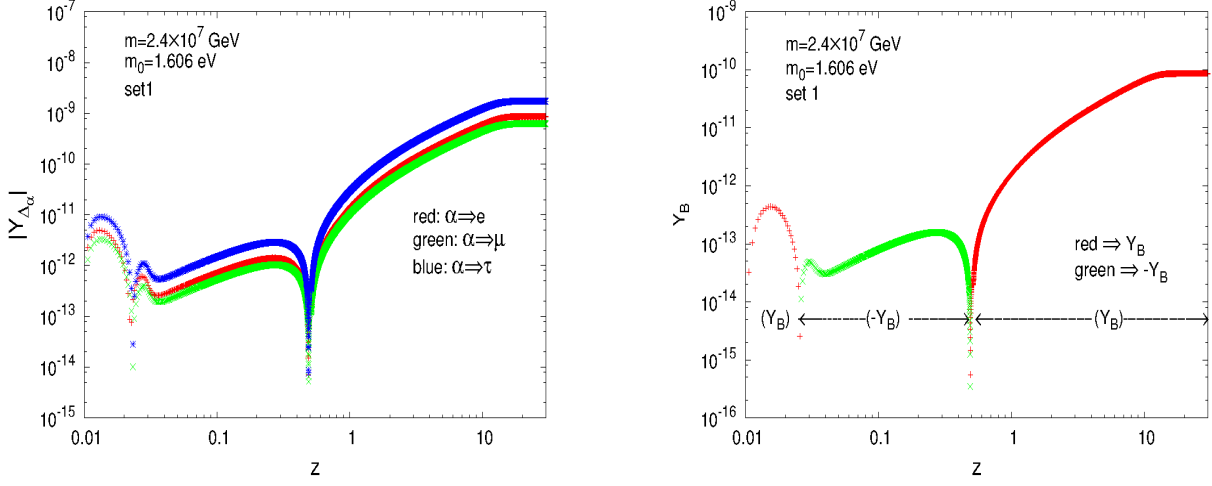


Figure 1: (colour online) Plot of flavor asymmetries(Y_{Δ_α}) (left) and baryon asymmetry(Y_B) (right) with z for a definite value of m and m_0 . In the Y_B vs z plot Y_B freezes at a value 8.67×10^{-11} (which is just midway between the the experimental bound).

From Table 3 it is clear that we can get a bound on the right handed neutrino mass (m) using the Y_B constraint $(8.55 < Y_B \times 10^{11} < 8.77)^{10}$ for each value of m_0 (rather for each value of the set $\{p, q, \alpha, \beta, m_0\}$). Similarly we plot (Fig.2) the final value of Y_B with the mass of the lowest right handed neutrino (m) for five different values of m_0 . In each of these plots we draw two lines parallel to abscissa, one at $Y_B = 8.55 \times 10^{-11}$ and the other at $Y_B = 8.77 \times 10^{-11}$. The value of m where the lines meet the Y_B vs m curve give the lower and upper bound on m respectively. Allowed range of m for different m_0 s are tabulated in Table 5. It is to be noticed that after imposition of baryon asymmetry bound the sign of the phase parameters and the the mass of the right handed neutrinos get a restriction and the fully constrained (by both oscillation data and baryon asymmetry bound) parameter space is presented in Table 6.

¹⁰or equivalently the bound on η_B is $(6.02 < \eta_B \times 10^{10} < 6.18)$ [7].

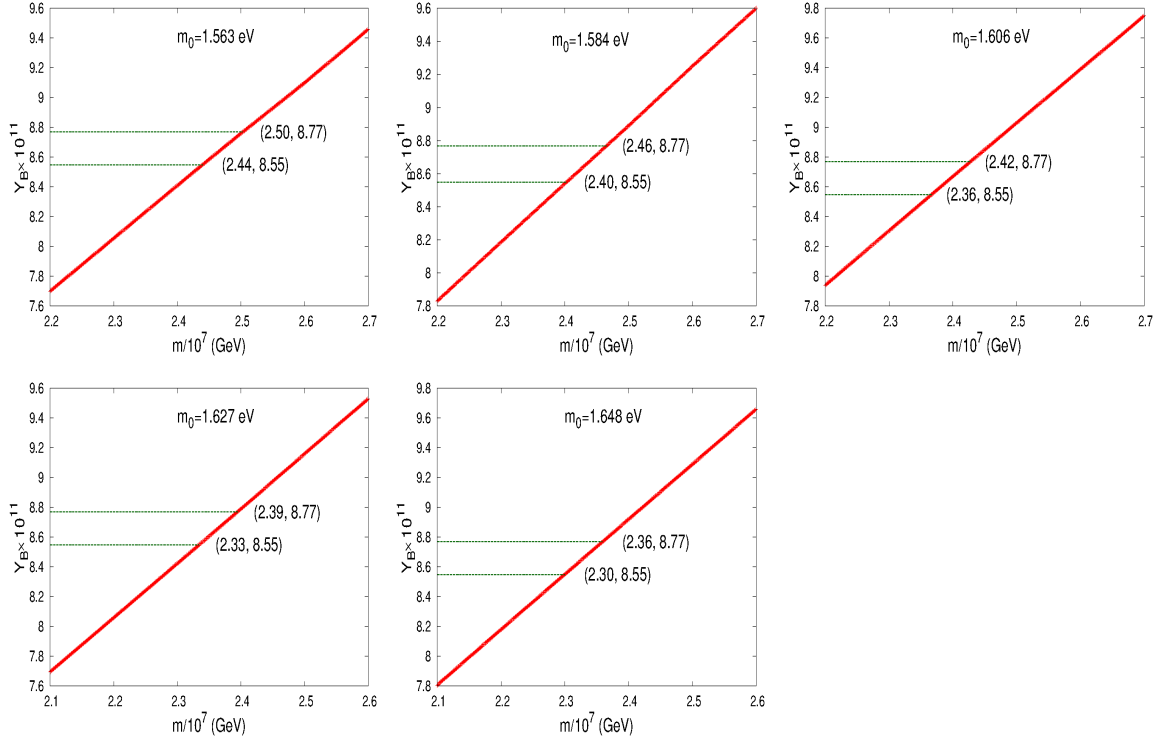


Figure 2: (colour online) Plot of Final Y_B with m for different values of m_0

Table 5: Range of m allowed by Y_B constraint for different m_0 values

$m_0 \times 10^9$ (GeV)	1.563	1.584	1.606	1.627	1.648
$\frac{m}{10^7}$ (GeV)	2.44 – 2.50	2.40 – 2.46	2.36 – 2.42	2.33 – 2.39	2.30 – 2.36

In the **τ -flavored** case the mass of the lightest right handed neutrino is less than 10^{12} GeV but greater than 10^9 GeV. In this regime we can not distinguish between e and μ flavors, whereas the τ flavor is decoupled. In this case the Boltzmann equations (3.17, 3.20) have to be solved for two flavors only, first one is for combined effect of e , μ and the second one is for τ . To find the combined asymmetry $Y_{\Delta_2}(= Y_{\Delta_e} + Y_{\Delta_\mu})$ we have to replace the α dependent terms in the RHS of eq.(3.20) by the sum over 2 flavors e , μ and Y_{Δ_τ} can be obtained simply by solving eq.(3.20) for τ flavor. Once we get $Y_{\Delta_2}(z)$ and $Y_{\Delta_\tau}(z)$ the frozen value of the baryon asymmetry scaled by entropy density Y_B can be calculated easily following the steps given in subsection 4.0.2. In this case too we compute variation of Y_B with z for ten different values of the set $\{p, q, \alpha, \beta, m_0\}$ (Table 2) and the set 2 of $\{p, q, \alpha, \beta\}$ is discarded due to the same argument as discussed in the **fully flavored** case. Final value of the ratio of baryon asymmetry to entropy density (Y_B) is calculated using set 1 of

Table 6: Sets of parameters allowed by both oscillation data and baryon asymmetry bound for case(i) of symmetry breaking with $\epsilon = -0.004$ (In fully flavored regime)

sets	parameters					
	p	q	α (deg.)	β (deg.)	$m_0 \times 10^9$ (GeV)	$\frac{m}{10^7}$ (GeV)
1	0.97	0.89	-113.5	120.5	1.563	2.44 – 2.50
					1.584	2.40 – 2.46
					1.606	2.36 – 2.42
					1.627	2.33 – 2.39
					1.648	2.30 – 2.36

$\{p, q, \alpha, \beta\}$ with five different values of m_0 for various values of right handed neutrino mass (m) in the range ($10^9 \text{ GeV} < m < 10^{12} \text{ GeV}$). It is found that Y_B in each of the combinations is far above the experimental upper bound.

Finally for the **unflavored** case the mass of the lightest right handed neutrino is greater than 10^{12} GeV and we can not differentiate between e, μ and τ flavors. For this case the Boltzmann equation for lepton flavor asymmetry is flavor independent. The flavor index dependent quantities in the RHS of the Boltzmann equation are replaced by the sum over three flavors (e, μ, τ). Solving the Boltzmann equation we get $Y_\Delta(z)$ from which the baryon asymmetry parameter (Y_B) can be computed using the formulas given in subsection 4.0.3. In spite of maximum value of CP asymmetry parameters in this regime the final value of lepton flavor asymmetry turns out to be zero. The reason behind this anomalous behaviour is discussed below.

The cyclic symmetry invariant m_D and $M_R = \text{diag}(m, m, m + \epsilon)$ dictates the CP asymmetry parameters ($\varepsilon_i = \sum_\alpha \varepsilon_i^\alpha$) as $\varepsilon_1 = -\varepsilon_2$ and $\varepsilon_3 = 0$ (detailed calculation shown in appendix A.1). Now looking at second Boltzmann equation (3.20) it is clear that at the starting point of iteration the second term in the RHS is zero since we have taken the initial condition ¹¹ $Y_\Delta(z = 0) = 0$. So $\frac{dY_\Delta}{dz}$ consists of sum of three terms involving $\varepsilon_1, \varepsilon_2$ and ε_3 respectively. The term containing ε_3 has null contribution whereas the terms involving ε_1 and ε_2 cancels each other¹² as a result we get $\frac{dY_\Delta}{dz} = 0$. So generation of lepton asymmetry as well as baryon asymmetry is not possible in this unflavored

¹¹It is to be noted that second term of eq.(3.20) is basically the washout term which tends to erase any pre-existing asymmetry. Thus even if we start with some pre-existing asymmetry as initial condition this second term will have no effect other than diminishing that asymmetry.

¹²Coefficients of ε_1 and ε_2 differ only by the parameter m_{N_i} (mass of the right handed neutrino of corresponding generation), i.e apart from identical common factors ε_1 contains m_{N_1} whereas that of ε_2 contains m_{N_2} . But in our breaking scheme ($M_R = \text{diag}(m, m, m + \epsilon)$) $\Rightarrow m_{N_1} = m_{N_2}$. Therefore the terms involving ε_1 and ε_2 in the second Boltzmann equation are exactly same and thus cancels each other when $\varepsilon_1 = -\varepsilon_2$.

leptogenesis scenario.

As a result of imposing baryon asymmetry bound together with neutrino oscillation data, the right handed neutrino mass scale is restricted and among set 1 and set 2, only the parameters belonging to set 1 are selected. Hence the value of the Dirac CP phase δ_{CP} is fixed to 29.38° .

5.2 Numerical analysis for case(ii) of symmetry breaking

As mentioned in eq.(2.8) for this case the symmetry breaking parameter is introduced in the ‘22’ element of M_R . The mixing angles and mass eigenvalues are calculated using the resulting mass matrix given in eq.(2.10). In this case the lowest allowed value of the breaking parameter ϵ is -0.004 and for this value of ϵ we get 16 allowed values of the parameter set $\{p, q, \alpha, \beta, m_0\}$. We have six different sets of $\{p, q, \alpha, \beta\}$ and for set 1 and set 2 of $\{p, q, \alpha, \beta\}$ there are six different m_0 values whereas set 3-6 has one allowed m_0 value each. Thus in total we have $16(= 6 + 6 + 1 + 1 + 1 + 1)$ values for the set of parameters $\{p, q, \alpha, \beta, m_0\}$ which are tabulated in Table 7. In this case (case(ii)) too normal hierarchy is preferred but θ_{23} is now selected in the 2nd octant ($48.07^\circ - 49.09^\circ$). The sign of α and β remain unsettled between the following sets: (1 and 2), (3 and 4), (5 and 6). These in effect again produce sign ambiguity in Dirac CP phase: $\delta_{\text{CP}} = 44.45^\circ$ for the set 1 with $(\alpha = -125^\circ, \beta = 116^\circ)$ and $\delta_{\text{CP}} = -44.45^\circ$ for the set 2 with $(\alpha = 125^\circ, \beta = -116^\circ)$, $\delta_{\text{CP}} = 16.56^\circ$ for the set 3 with $(\alpha = -121^\circ, \beta = 113.5^\circ)$ and $\delta_{\text{CP}} = -16.56^\circ$ for the set 4 with $(\alpha = 121^\circ, \beta = -113.5^\circ)$, $\delta_{\text{CP}} = 14.27^\circ$ for the set 5 with $(\alpha = -117.5^\circ, \beta = 116^\circ)$ and $\delta_{\text{CP}} = -14.27^\circ$ for the set 6 with $(\alpha = 117.5^\circ, \beta = -116^\circ)$. To solve the sign ambiguity of the phases and to restrict the right handed neutrino mass scale m for this case also, we investigate all three subcases of leptogenesis namely, fully flavored, τ -flavored and unflavored.

Following the same steps as the previous case (case(i) of symmetry breaking) in **fully flavored** regime evolution of Y_B with z is computed with each of the 16 values of the set $\{p, q, \alpha, \beta, m_0\}$ for different values of right handed neutrino mass m . After the numerical analysis of each of the above combinations it is found that for set 2, 4, 6 of $\{p, q, \alpha, \beta\}$ Y_B produced at high z value attain a fixed negative value and thus these three sets can be discarded. Therefore we are left with set 1, 3, 5 of $\{p, q, \alpha, \beta\}$ and their corresponding m_0 values (six for set 1 and one each for set 3 and set 5) which gives rise to constant positive Y_B at high z . The final value of ratio of baryon asymmetry to entropy density (Y_B) for each set of values with different m values are presented in Table 8. As an example we choose the case ($m = 4.4 \times 10^7$ GeV, $m_0 = 1.933 \times 10^{-9}$ GeV) of set 1 and show the evolution of flavor asymmetries (Y_{Δ_α}) and baryon asymmetry (Y_B) with z in Fig.3.

Table 7: Sets of parameters allowed by oscillation data for case(ii) of symmetry breaking with $\epsilon = -0.004$

sets	parameters					
	p	q	α (deg.)	β (deg.)	$m_0 \times 10^9$ (GeV)	$\frac{m}{10^7}$ (GeV)
1	0.97	1.05	-125	116	1.881	unrestricted for all m_0 values
					1.907	
					1.933	
					1.959	
					1.984	
					2.009	
2	0.97	1.05	125	-116	1.881	unrestricted for all m_0 values
					1.907	
					1.933	
					1.959	
					1.984	
					2.009	
3	0.89	0.95	-121	113.5	1.717	unrestricted
4	0.89	0.95	121	-113.5	1.717	unrestricted
5	0.91	0.91	-117.5	116	1.683	unrestricted
6	0.91	0.91	117.5	-116	1.683	unrestricted

The sign of the different flavor asymmetry parameters at various z values are shown in Table 9¹³ and in this case too the numerical analysis reveals that $|Y_{\Delta\tau}| > |Y_{\Delta e} + Y_{\Delta\mu}|$.

To get a bound on right handed neutrino mass we plot in Fig.4 and Fig.5 the final value of Y_B with m for different values of m_0 (or rather for different values of the parameter set $\{p, q, \alpha, \beta, m_0\}$) showing the allowed region and the range of m thus obtained are presented in Table 10 and the finally surviving parameter space (after imposing the baryon asymmetry bound) is shown in Table 11.

¹³It is to be noted that the table is for set 1 only which survives the baryon asymmetry bound. The sign of the corresponding parameters belonging to set 2 (complex conjugate of set 1) will have a relative negative sign.

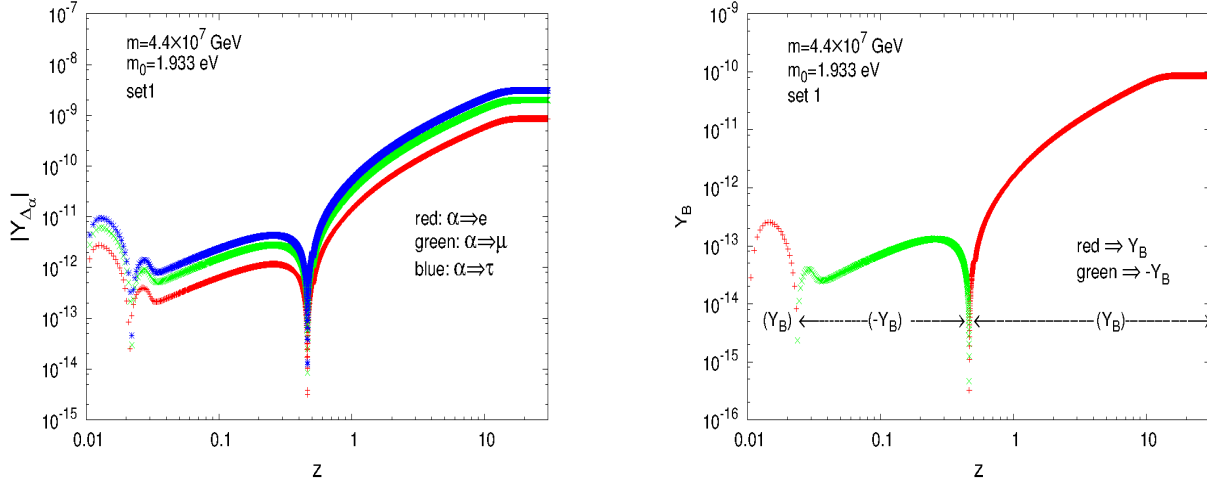


Figure 3: (colour online) Plot of flavor asymmetries(Y_{Δ_α}) (left), baryon asymmetry(Y_B) (right) with z for a definite value of m and m_0 . In Y_B vs z plot Y_B freezes at a value 8.60×10^{-11} .

Table 8: Final value of baryon asymmetry for set 1, 3, 5 of $\{p, q, \alpha, \beta\}$ (m and m_0 are in GeV)

$m_0 \times 10^9$	$\frac{m}{10^7}$	$Y_B \times 10^{11}$								
		1.3	1.5	1.7	2.0	4.2	4.4	4.6	4.8	
1.881		2.46	2.84	3.22	3.79	7.96	8.34	8.72	9.10	set 1
1.907		2.50	2.89	3.27	3.85	8.08	8.47	8.85	9.24	
1.933		2.54	2.93	3.32	3.91	8.21	8.60	8.99	9.38	
1.959		2.57	2.97	3.37	3.96	8.33	8.72	9.12	9.52	
1.984		2.61	3.01	3.42	4.02	8.45	8.85	9.25	9.65	
2.009		2.65	3.06	3.46	4.08	8.56	8.97	9.38	9.79	
1.717		7.54	8.70	9.86	11.61	24.38	25.54	26.70	27.86	set 3
1.683		6.15	7.09	8.04	9.46	19.87	20.81	21.76	22.70	set 5

Table 9: signs of different asymmetries at different z values

	$z = 0.01 \rightarrow 0.02$				$z = 0.02 \rightarrow 0.46$				$z > 0.46$			
	Y_{Δ_e}	Y_{Δ_μ}	Y_{Δ_τ}	Y_B	Y_{Δ_e}	Y_{Δ_μ}	Y_{Δ_τ}	Y_B	Y_{Δ_e}	Y_{Δ_μ}	Y_{Δ_τ}	Y_B
set 1	-ve	-ve	+ve	+ve	+ve	+ve	-ve	-ve	-ve	-ve	+ve	+ve

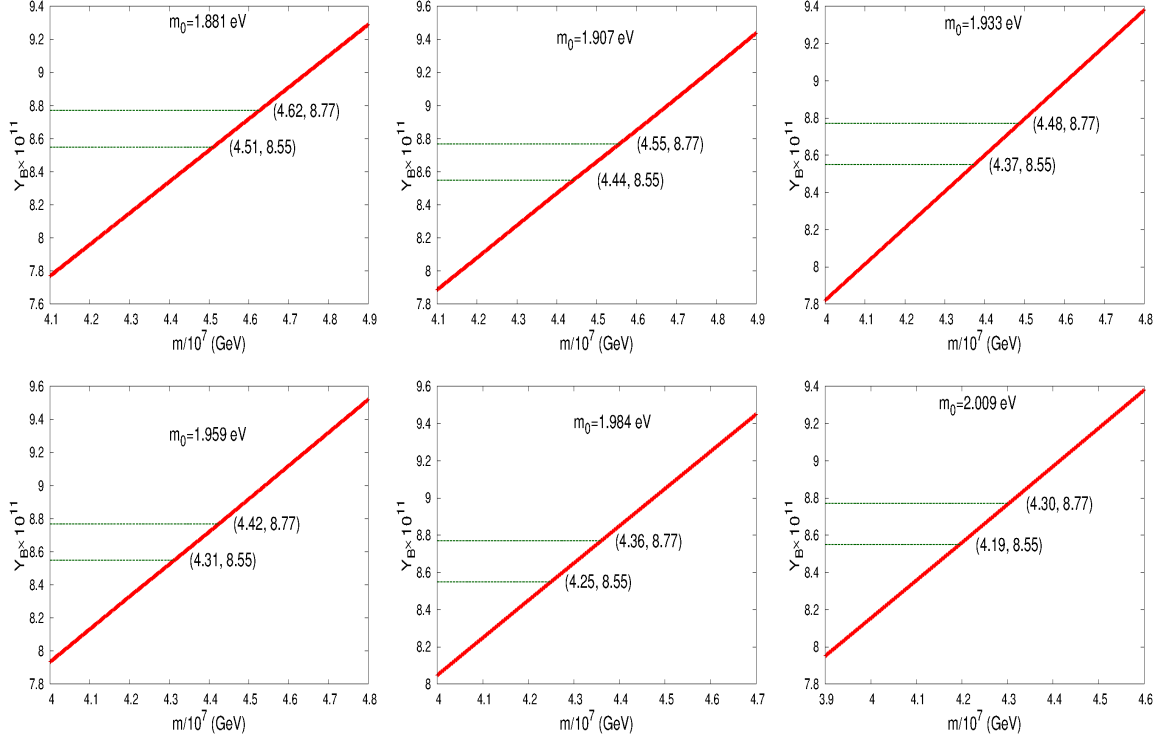


Figure 4: (colour online) Plot of Final Y_B with m for different values of m_0 with set 1 of $\{p, q, \alpha, \beta\}$

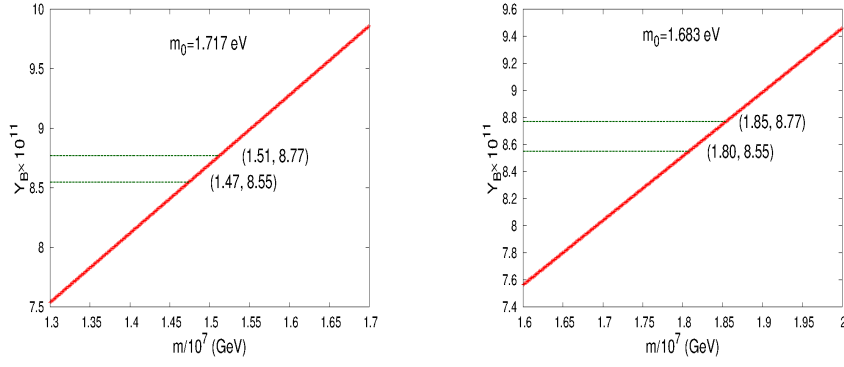


Figure 5: (colour online) Plot of Final Y_B with m for different values of m_0 with set 3 (left) and set 5 (right) of $\{p, q, \alpha, \beta\}$

In τ -**flavored** regime we encounter exactly the same consequences as in the previous case (case(i) of symmetry breaking), i.e the final value of baryon asymmetry (Y_B) produced for any value of right handed neutrino mass (in the range $10^9 < m(\text{GeV}) < 10^{12}$) with any of the 16 value of the set $\{p, q, \alpha, \beta, m_0\}$ is far beyond the the experimental upper bound.

Table 10: Range of m allowed by Y_B constraint for different m_0 values with set 1, 3, 5 of $\{p, q, \alpha, \beta\}$

	set 1						set 3	set 5
$m_0 \times 10^9$ (GeV)	1.881	1.907	1.933	1.959	1.984	2.009	1.717	1.683
$\frac{m}{10^7}$ (GeV)	4.51 – 4.62	4.44 – 4.55	4.37 – 4.48	4.31 – 4.42	4.25 – 4.36	4.19 – 4.30	1.47 – 1.51	1.80 – 1.85

Table 11: Sets of parameters allowed by both oscillation data and baryon asymmetry bound for case(ii) of symmetry breaking with $\epsilon = -0.004$ (In fully flavored regime)

	parameters					
sets	p	q	α (deg.)	β (deg.)	$m_0 \times 10^9$ (GeV)	$\frac{m}{10^7}$ (GeV)
1	0.97	1.05	-125	116	1.881	4.51 – 4.62
					1.907	4.44 – 4.55
					1.933	4.37 – 4.48
					1.959	4.31 – 4.42
					1.984	4.25 – 4.36
					2.009	4.19 – 4.30
3	0.89	0.95	-121	113.5	1.717	1.47 – 1.51
5	0.91	0.91	-117.5	116	1.683	1.80 – 1.85

In **Unflavored** regime the required CP asymmetry parameters ($\varepsilon_i = \sum_{\alpha} \varepsilon_i^{\alpha}$) for cyclic symmetric m_D and $M_R = \text{diag}(m, m + \epsilon, m)$ comes out to be $\varepsilon_2 = 0$ and $\varepsilon_1 = -\varepsilon_3$ (detailed calculation shown in appendix A.2). Therefore using the same argument as the previous case (case(i) of symmetry breaking) it can be shown that at the starting point of iteration $\frac{dY_{\Delta}}{dz} = 0$ and hence baryogenesis through leptogenesis is not possible in this regime.

We conclude the analysis of case(ii) with a remark that imposition of baryon asymmetry bound together with neutrino oscillation data constrains the right handed neutrino mass scale as well as selects only those parameters belonging to set 1, 3, 5. Hence the value of δ_{CP} can only be positive as: 44.45° for set 1, 16.56° for set 3 and 14.27° for set 5.

5.3 Numerical analysis for case(iii) of symmetry breaking

This variant of symmetry breaking arises due to incorporation of the breaking parameter in the ‘11’ element of M_R and the neutrino physics observable (mass squared differences and mixing angles) are calculated with the m_{ν} matrix given in eq.(2.11). In this case too the lowest allowed

value of breaking parameter (ϵ) is -0.004 for which the parameter space allowed by oscillation data is presented in Table 12. For the case (iii) normal hierarchy is preferred and θ_{23} is selected

Table 12: Sets of parameters allowed by oscillation data for case(iii) of symmetry breaking with $\epsilon = -0.004$

sets	parameters					
	p	q	α (deg.)	β (deg.)	$m_0 \times 10^9$ (GeV)	$\frac{m}{10^7}$ (GeV)
1	0.91	0.91	-116	117.5	1.695 1.717 1.739 1.761	unrestricted for all m_0 values
2	0.91	0.91	116	-117.5	1.695 1.717 1.739 1.761	unrestricted for all m_0 values
3	0.95	0.89	-113.5	121	1.717	unrestricted
4	0.95	0.89	121	-113.5	1.717	unrestricted
5	1.05	0.97	-116	125	1.881 1.907 1.933 1.959 1.984 2.009	unrestricted for all m_0 values
6	1.05	0.97	116	-125	1.881 1.907 1.933 1.959 1.984 2.009	unrestricted for all m_0 values

in the 1st octant ($40.90^\circ - 41.92^\circ$). The sign of α and β remain unsettled between the following sets: (1 and 2), (3 and 4), (5 and 6). These in effect again produce sign ambiguity in Dirac CP phase: $\delta_{\text{CP}} = 14.41^\circ$ for the set 1 with ($\alpha = -116^\circ$, $\beta = 117.5^\circ$) and $\delta_{\text{CP}} = -14.41^\circ$ for the set 2 with ($\alpha = 116^\circ$, $\beta = -117.5^\circ$), $\delta_{\text{CP}} = 16.36^\circ$ for the set 3 with ($\alpha = -113.5^\circ$, $\beta = 121^\circ$) and $\delta_{\text{CP}} = -16.36^\circ$ for the set 4 with ($\alpha = 113.5^\circ$, $\beta = -121^\circ$), $\delta_{\text{CP}} = 44.41^\circ$ for the set 5 with

($\alpha = -116^\circ$, $\beta = 125^\circ$) and $\delta_{\text{CP}} = -44.41^\circ$ for the set 6 with ($\alpha = 116^\circ$, $\beta = -125^\circ$).

To solve the sign ambiguity of the phases and to restrict the right handed neutrino mass scale m for this case also, we proceed to calculate the baryon asymmetry with these 22 values¹⁴ of the set of parameters $\{p, q, \alpha, \beta, m_0\}$. Following exactly the same procedure as done in the previous cases we calculate the final value of baryon asymmetry for all three sub categories of leptogenesis namely **fully flavored**, **τ -flavored** and **unflavored**.

The result obtained in the **fully flavored case** is analogous to that we have got in case(i) and case(ii). In this case set 2, 4 and 6 generate constant negative value of Y_B at high z and thus those sets are discarded. It is found that set 1, 3 and 5 are able to produce a Y_B that freezes to a positive value at low temperature. The final value of baryon asymmetry parameter produced by them for different values of the right handed neutrino mass m is shown in the Table 13. As an example we choose the case ($m = 1.7 \times 10^7$ GeV, $m_0 = 1.761 \times 10^{-9}$ GeV) of set 1 and show the evolution of flavor asymmetries (Y_{Δ_α}) and baryon asymmetry (Y_B) with z in Fig. 6. The sign of different flavor asymmetry parameters for this chosen example is given in Table 14 and one important outcome of numerical analysis in this case is $|Y_{\Delta_e}| < |Y_{\Delta_\mu} + Y_{\Delta_\tau}|$. The plots of Y_B vs m to get a bound on right handed neutrino mass are given in Figures 7, 8, 9 and the bound obtained from those plots are tabulated clearly in Table 15. Finally, the fully constrained parameter space for case(iii) of symmetry breaking is presented in Table 16.

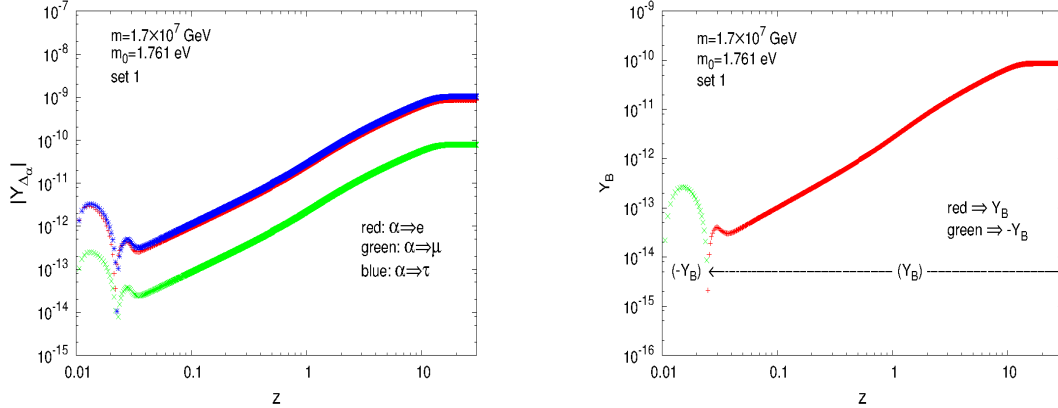


Figure 6: (colour online) Plot of flavor asymmetries(Y_{Δ_α}) (left), baryon asymmetry(Y_B) (right) with z for a definite value of m and m_0 . In Y_B vs z plot Y_B freezes at a value 8.68×10^{-11} .

¹⁴22=4(set 1) +4(set 2) +1(set 3) +1(set 4) +6(set 5) +6(set 6).

Table 13: Final value of baryon asymmetry for set 1, 3, 5 of $\{p, q, \alpha, \beta\}$ (m and m_0 are in GeV)

$m_0 \times 10^9$ \ $\frac{m}{10^7}$	$Y_B \times 10^{11}$								
	1.3	1.5	1.7	2.0	4.0	4.3	4.6	5.0	
1.695	6.36	7.34	8.32	9.79	19.58	21.04	22.51	24.47	set 1
1.717	6.45	7.45	8.44	9.93	19.86	21.35	22.84	24.83	
1.739	6.55	7.55	8.56	10.07	20.15	21.66	23.17	25.19	
1.761	6.64	7.66	8.68	10.21	20.43	21.96	23.50	25.54	
1.717	7.74	8.94	10.13	11.92	23.84	25.62	27.41	29.80	set 3
1.881	2.53	2.92	3.31	3.89	7.78	8.37	8.95	9.73	set 5
1.907	2.57	2.96	3.36	3.95	7.91	8.50	9.09	9.88	
1.933	2.60	3.01	3.41	4.01	8.03	8.63	9.23	10.03	
1.959	2.64	3.05	3.46	4.07	8.14	8.75	9.37	10.18	
1.984	2.68	3.10	3.51	4.13	8.26	8.88	9.50	10.33	
2.009	2.72	3.14	3.56	4.19	8.38	9.00	9.63	10.47	

Table 14: signs of different asymmetries at different z values

	$z = 0.01 \rightarrow 0.02$				$z > 0.02$			
	Y_{Δ_e}	Y_{Δ_μ}	Y_{Δ_τ}	Y_B	Y_{Δ_e}	Y_{Δ_μ}	Y_{Δ_τ}	Y_B
set 6	+ve	-ve	-ve	-ve	-ve	+ve	+ve	+ve

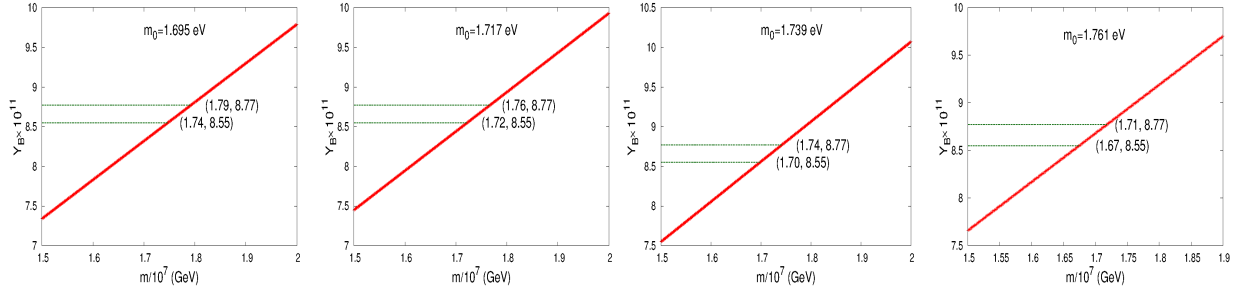


Figure 7: (colour online) Plot of Final Y_B with m for different values of m_0 with set 1 of $\{p, q, \alpha, \beta\}$

Results of τ -flavored and unflavored case are same as that of case(i) and (ii). In τ -flavored regime set 1, 3 and 5 generate a positive value of baryon asymmetry which is far beyond the

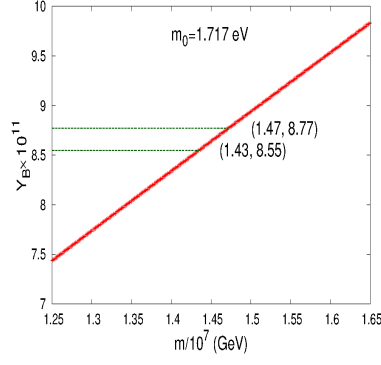


Figure 8: (colour online) Plot of Final Y_B with m for different values of m_0 with set 3 of $\{p, q, \alpha, \beta\}$

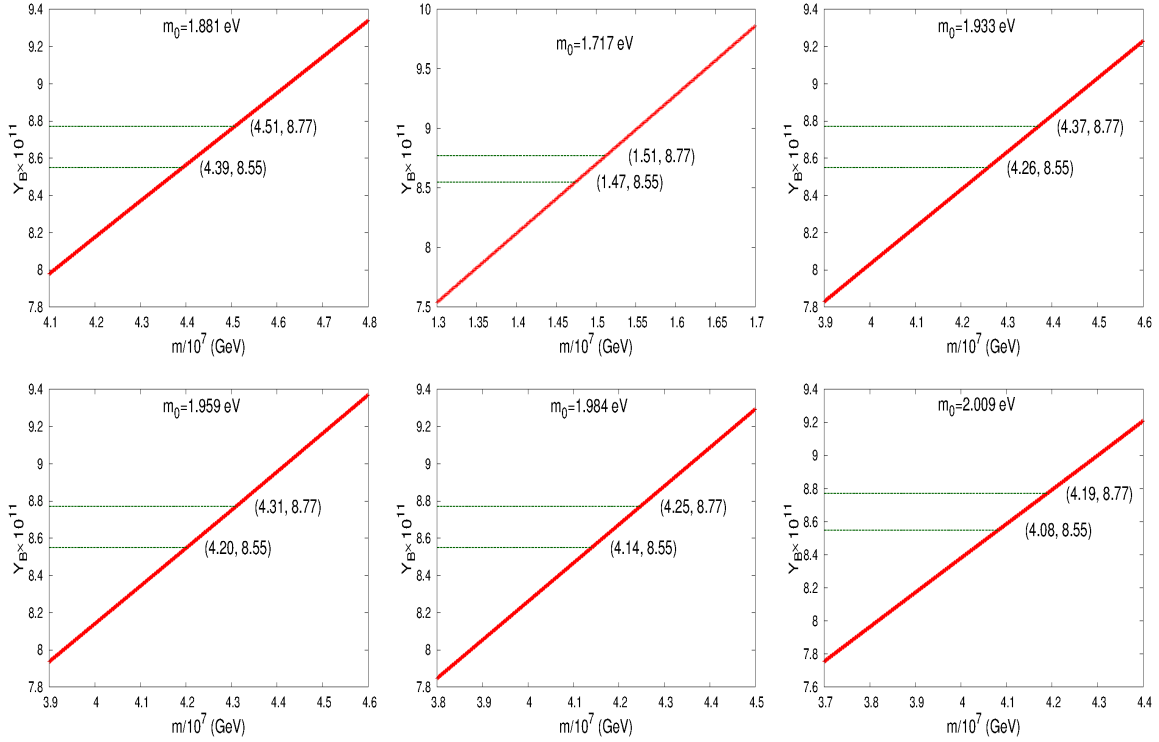


Figure 9: (colour online) Plot of Final Y_B with m for different values of m_0 with set 5 of $\{p, q, \alpha, \beta\}$

experimental upper limit, whereas in the **unflavored** regime¹⁵ generation of baryon asymmetry is

¹⁵ CP asymmetry parameters ($\varepsilon_i = \sum_{\alpha} \varepsilon_i^{\alpha}$) for cyclic symmetric m_D and $M_R = \text{diag}(m + \epsilon, m, m)$ comes out to be $\varepsilon_1 = 0$ and $\varepsilon_2 = -\varepsilon_3$ (detailed calculation shown in appendix A.3). Therefore using the same argument as the previous case (case(i) of symmetry breaking) it can be shown that at the starting point of iteration $\frac{dY_{\Delta}}{dz} = 0$ and

Table 15: Range of m allowed by Y_B constraint for different m_0 values with set 1, 3, 5 of $\{p, q, \alpha, \beta\}$

	set 1				set 3	set 5					
$m_0 \times 10^9$ (GeV)	1.695	1.717	1.739	1.761	1.717	1.881	1.907	1.933	1.959	1.984	2.009
$\frac{m}{10^7}$ (GeV)	1.74– 1.79	1.72– 1.76	1.70– 1.74	1.67– 1.71	1.43– 1.47	4.39– 4.51	4.32– 4.43	4.26– 4.37	4.20 4.31	4.14– 4.25	4.08– 4.19

Table 16: Sets of parameters allowed by both oscillation data and baryon asymmetry bound for case(iii) of symmetry breaking with $\epsilon = -0.004$ (In fully flavored regime)

	parameters					
sets	p	q	α (deg.)	β (deg.)	$m_0 \times 10^9$ (GeV)	$\frac{m}{10^7}$ (GeV)
1	0.91	0.91	-116	117.5	1.695	1.74 – 1.79
					1.717	1.72 – 1.76
					1.739	1.70 – 1.74
					1.761	1.67 – 1.71
3	0.95	0.89	-121	113.5	1.717	1.43 – 1.47
5	1.05	0.97	-116	125	1.881	4.39 – 4.51
					1.907	4.32 – 4.43
					1.933	4.26 – 4.37
					1.959	4.20 – 4.31
					1.984	4.14 – 4.25
					2.009	4.08 – 4.19

not at all possible.

Again the baryon asymmetry bound along with oscillation data admits only those parameters belonging to set 1, 3, 5 as well as constrains mass scale of the right handed neutrino. Hence in this case also δ_{CP} can have only positive values: 14.41° for set 1, 16.36° for set 3 and 41.41° for set 5.

It is evident from the numerical analysis of the three cases (case(i), (ii) and (iii)) that baryon asymmetry in the allowed range can only be generated in fully flavored regime. For τ -flavored regime all three cases produce excess baryon asymmetry. The phases α, β are restricted by light

hence generation of baryon asymmetry is not possible.

neutrino data and they are not closer to 0 or π also. This τ -flavored regime is closer to the resonant enhancement region and phase suppression does not occur. So, it goes beyond experimental range. For unflavored regime in all three cases our breaking mechanism of cyclic symmetry and summation over flavor produce null contribution to the lepton asymmetry and hence baryon asymmetry although the resonant enhancement of ε_i^α 's occur in this regime.

6 Summary

We consider an $SU(2)_L \times U(1)_Y$ model with three right chiral neutrinos invoking type-I seesaw mechanism and cyclic symmetry in the neutrino sector. Since, the symmetry invariant model generates two fold degeneracy in the light neutrino mass, the model forbids to determine three mixing angles in an unique way as well as generates vanishing value of one mass squared difference. A possible way to get rid of those shortcomings is due to the breaking of the cyclic symmetry imposed. Symmetry breaking is incorporated in a minimal way through one small breaking parameter (ϵ). Armed with such modifications, we apply the most general diagonalization method to find out mass eigenvalues, mixing angles and Dirac CP phase. First we restrict the parameter space by fixing the neutrino oscillation experimental data. The first level of restriction is done for three different cases of symmetry breaking, viz (i) $M_R = \text{diag}\{m, m, m(1 + \epsilon)\}$, (ii) $M_R = \text{diag}\{m, m(1 + \epsilon), m\}$ and (iii) $M_R = \text{diag}\{m(1 + \epsilon), m, m\}$. We have seen that normal hierarchy of light neutrino masses is preferred for all three cases. The obtained parameter space prefer θ_{23} to be in the first octant ($\simeq 37^\circ$) for case (i), 2nd octant ($48^\circ - 49^\circ$) for case (ii) and 1st octant ($40^\circ - 41^\circ$) for case (iii). Furthermore the sign ambiguity in α and β produce both positive and negative values of δ_{CP} prior to the application of baryon asymmetry bound.

Next we investigate explicitly the effect of quasi degeneracy of right handed neutrino masses in enhancement of CP asymmetry. Among the allowed values of the set of Lagrangian parameters we choose only those sets corresponding to the lowest value of breaking parameter. Only those values of the parameter sets are used in calculation of leptogenesis. The phenomena of leptogenesis is studied in three different energy regimes $\{(m(\text{GeV}) < 10^9), (10^9 < m(\text{GeV}) < 10^{12}), (m(\text{GeV}) > 10^{12})\}$ where lepton flavors are fully distinguishable, partly distinguishable or indistinguishable respectively. Calculation of lepton asymmetry in these regimes are carried out thereafter solving the detailed set of Boltzmann equations. These lepton asymmetries are then converted to baryon asymmetry using suitable formulas.

Notable outcomes of our numerical analysis are:

- Only fully flavored leptogenesis is able to produce baryon asymmetry in the observed range. Unflavored leptogenesis is unable to generate any asymmetry in all the cases. τ -flavored leptogenesis although analytically allowed however numerical estimation shows that value of produced asymmetry is far beyond the present experimental limit.
- Using the cut on Y_B we have obtained a bound on right handed heavy neutrino mass $(1.43 - 4.62) \times 10^7$ GeV (considering all the cases (case(i), case(ii) and case(iii)) of fully flavored regime) which were unconstrained even after the restriction by neutrino oscillation data.
- Dirac CP phase takes only positive value in the range $14^\circ - 45^\circ$ after imposition of baryon asymmetry bound (considering all three cases of symmetry breaking).

A Appendix

A.1 CP asymmetry parameters in unflavored regime with case(i) of symmetry breaking ($M_R = \text{diag}\{m, m, m(1 + \epsilon)\}$)

The flavor summed CP asymmetry parameter relevant in this regime is given by $\varepsilon_i = \sum_{\alpha} \varepsilon_i^{\alpha}$:

$$\varepsilon_i = \frac{1}{4\pi v^2 H_{ii}} \sum_{j \neq i} \text{Im}\{H_{ij}^2\} g(x_{ij}) \quad (\text{A.1})$$

where $g(x_{ij}) = f(x_{ij}) + \frac{\sqrt{x_{ij}(1-x_{ij})}}{(1-x_{ij})^2 + \frac{H_{jj}^2}{16\pi^2 v^4}}$ and $x_{ij} = \frac{m_{N_j}^2}{m_{N_i}^2}$.

In this breaking scheme we have

$$x_{12} = 1, \quad x_{23} = x_{13} = (1 + \epsilon)^2 \quad \text{and} \quad x_{ji} = \frac{1}{x_{ij}}. \quad (\text{A.2})$$

The elements of H matrix (eq.(5.2)) are obtained as

$$\begin{aligned} H_{12} &= H_{23} = H_{31} = mm_0 Y = r e^{i\theta} \\ H_{13} &= H_{21} = H_{32} = mm_0 Y^* = r e^{-i\theta}. \end{aligned} \quad (\text{A.3})$$

Using these values we get

$$\begin{aligned} \varepsilon_1 &= \frac{1}{4\pi v^2 H_{11}} [\text{Im}\{H_{12}^2\} g(x_{12}) + \text{Im}\{H_{13}^2\} g(x_{13})] \\ &= \frac{1}{4\pi v^2 H_{11}} [r^2 \sin 2\theta g(1) - r^2 \sin 2\theta g((1 + \epsilon)^2)], \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned}
\varepsilon_2 &= \frac{1}{4\pi v^2 H_{22}} [Im\{H_{21}^2\}g(x_{21}) + Im\{H_{23}^2\}g(x_{23})] \\
&= \frac{1}{4\pi v^2 H_{22}} [-r^2 \sin 2\theta g(1) + r^2 \sin 2\theta g((1+\epsilon)^2)].
\end{aligned} \tag{A.5}$$

Therefore it is clear that $\varepsilon_1 = -\varepsilon_2$ since in our model $H_{11} = H_{22} = H_{33}$. Finally

$$\begin{aligned}
\varepsilon_3 &= \frac{1}{4\pi v^2 H_{33}} [Im\{H_{31}^2\}g(x_{31}) + Im\{H_{32}^2\}g(x_{32})] \\
&= \frac{1}{4\pi v^2 H_{22}} [r^2 \sin 2\theta g\{\frac{1}{(1+\epsilon)^2}\} - r^2 \sin 2\theta g\{\frac{1}{(1+\epsilon)^2}\}] \\
&= 0
\end{aligned} \tag{A.6}$$

A.2 CP asymmetry parameters in unflavored regime with case(ii) of symmetry breaking ($M_R = diag\{m, m(1+\epsilon), m\}$)

In this breaking scheme we have

$$x_{12} = \frac{1}{x_{23}} = (1+\epsilon)^2, \quad x_{13} = 1 \quad \text{and} \quad x_{ji} = \frac{1}{x_{ij}}. \tag{A.7}$$

The elements of H matrix (eq.(5.2)) are obtained as

$$\begin{aligned}
H_{12} &= H_{23} = H_{31} = mm_0 Y = re^{i\theta} \\
H_{13} &= H_{21} = H_{32} = mm_0 Y^* = re^{-i\theta}.
\end{aligned} \tag{A.8}$$

Using these values we get

$$\begin{aligned}
\varepsilon_1 &= \frac{1}{4\pi v^2 H_{11}} [Im\{H_{12}^2\}g(x_{12}) + Im\{H_{13}^2\}g(x_{13})] \\
&= \frac{1}{4\pi v^2 H_{11}} [r^2 \sin 2\theta g((1+\epsilon)^2) - r^2 \sin 2\theta g(1)],
\end{aligned} \tag{A.9}$$

$$\begin{aligned}
\varepsilon_3 &= \frac{1}{4\pi v^2 H_{33}} [Im\{H_{31}^2\}g(x_{31}) + Im\{H_{32}^2\}g(x_{32})] \\
&= \frac{1}{4\pi v^2 H_{22}} [r^2 \sin 2\theta g(1) - r^2 \sin 2\theta g((1+\epsilon)^2)]
\end{aligned} \tag{A.10}$$

i.e we get $\varepsilon_1 = -\varepsilon_3$ and

$$\begin{aligned}
\varepsilon_2 &= \frac{1}{4\pi v^2 H_{22}} [Im\{H_{21}^2\}g(x_{21}) + Im\{H_{23}^2\}g(x_{23})] \\
&= \frac{1}{4\pi v^2 H_{22}} [-r^2 \sin 2\theta g\{\frac{1}{(1+\epsilon)^2}\} + r^2 \sin 2\theta g\{\frac{1}{(1+\epsilon)^2}\}] \\
&= 0
\end{aligned} \tag{A.11}$$

A.3 CP asymmetry parameters in unflavored regime with case(iii) of symmetry breaking ($M_R = \text{diag}\{m(1+\epsilon), m, m\}$)

In this breaking scheme we have

$$x_{12} = x_{13} = \frac{1}{(1+\epsilon)^2}, \quad x_{23} = 1 \quad \text{and} \quad x_{ji} = \frac{1}{x_{ij}}. \quad (\text{A.12})$$

The elements of H matrix (eq.(5.2)) are obtained as

$$\begin{aligned} H_{12} &= H_{23} = H_{31} = mm_0 Y = re^{i\theta} \\ H_{13} &= H_{21} = H_{32} = mm_0 Y^* = re^{-i\theta}. \end{aligned} \quad (\text{A.13})$$

Using these values we get

$$\begin{aligned} \varepsilon_1 &= \frac{1}{4\pi v^2 H_{11}} [Im\{H_{12}^2\}g(x_{12}) + Im\{H_{13}^2\}g(x_{13})] \\ &= \frac{1}{4\pi v^2 H_{11}} [r^2 \sin 2\theta g\{(1+\epsilon)^{-2}\} - r^2 \sin 2\theta g\{(1+\epsilon)^{-2}\}] \\ &= 0 \end{aligned} \quad (\text{A.14})$$

$$\begin{aligned} \varepsilon_2 &= \frac{1}{4\pi v^2 H_{22}} [Im\{H_{21}^2\}g(x_{21}) + Im\{H_{23}^2\}g(x_{23})] \\ &= \frac{1}{4\pi v^2 H_{22}} [-r^2 \sin 2\theta g\{(1+\epsilon)^2\} + r^2 \sin 2\theta g\{1\}] \end{aligned} \quad (\text{A.15})$$

$$\begin{aligned} \varepsilon_3 &= \frac{1}{4\pi v^2 H_{33}} [Im\{H_{31}^2\}g(x_{31}) + Im\{H_{32}^2\}g(x_{32})] \\ &= \frac{1}{4\pi v^2 H_{22}} [r^2 \sin 2\theta g\{(1+\epsilon)^2\} - r^2 \sin 2\theta g\{1\}] \end{aligned} \quad (\text{A.16})$$

i.e we get $\varepsilon_2 = -\varepsilon_3$.

References

- [1] V. A. Rubakov and M. E. Shaposhnikov, Usp. Fiz. Nauk **166**, 493 (1996) [Phys. Usp. **39**, 461 (1996)] [hep-ph/9603208].
- [2] M. Trodden, Rev. Mod. Phys. **71**, 1463 (1999) [hep-ph/9803479].
- [3] A. Riotto, [hep-ph/9807454].
- [4] J. M. Cline, [hep-ph/0609145].
- [5] M. Dine and A. Kusenko, Rev. Mod. Phys. **76**, 1 (2003) [hep-ph/0303065].
- [6] A.D. Sakharov, Zh. Eksp. Teor. Fiz. Pis'ma **5**, 32 (1967); JETP Lett. **91B**, 24 (1967).
- [7] P. S. B. Dev and R. N. Mohapatra, Phys. Rev. D **92**, no. 1, 016007 (2015) [arXiv:1504.07196 [hep-ph]].
- [8] P. A. R. Ade *et al.* [Planck Collaboration], [arXiv:1502.01589 [astro-ph.CO]].
- [9] P. A. R. Ade *et al.* [Planck Collaboration], Astron. Astrophys. **571**, A16 (2014) [arXiv:1303.5076 [astro-ph.CO]].
- [10] M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.
- [11] A. Riotto and M. Trodden, Ann. Rev. Nucl. Part. Sci. **49** (1999) 35 [arXiv:hep-ph/9901362].
- [12] M. Yu. Khlopov, *Cosmoparticle Physics*, World Scientific, Singapore (1999).
- [13] S. Davidson, E. Nardi and Y. Nir, Phys. Rept. **466**, 105 (2008) [arXiv:0802.2962 [hep-ph]].
- [14] W. Buchmuller and M. Plumacher, Int. J. Mod. Phys. A **15**, 5047 (2000) [hep-ph/0007176].
- [15] W. Bernreuther, Lect. Notes Phys. **591**, 237 (2002) [hep-ph/0205279].
- [16] D. V. Forero, M. Tortola and J. W. F. Valle, Phys. Rev. D **90**, no. 9, 093006 (2014) [arXiv:1405.7540 [hep-ph]].
- [17] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado and T. Schwetz, JHEP **1212**, 123 (2012) [arXiv:1209.3023 [hep-ph]].
- [18] D. V. Forero, M. Tortola and J. W. F. Valle, Phys. Rev. D **86** (2012) 073012 [arXiv:1205.4018 [hep-ph]].
- [19] B. Adhikary, M. chakraborty and A. Ghosal, JHEP **1310**, 043 (2013) Erratum: [JHEP **1409**, 180 (2014)] [arXiv:1307.0988 [hep-ph]].

- [20] Y. Koide, [hep-ph/0005137].
- [21] A. Damanik, M. Satriawan, P. Anggraita, A. Hermanto and Muslim, J. Theor. Comput. Stud. **8**, (2008) 0102 [arXiv:0710.1742 [hep-ph]].
- [22] A. Damanik, [arXiv:1004.1457 [hep-ph]].
- [23] R. Samanta and A. Ghosal, [arXiv:1507.02582 [hep-ph]].
- [24] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B **384**, 169 (1996) [hep-ph/9605319].
- [25] A. Pilaftsis and T. E. J. Underwood, Nucl. Phys. B **692**, 303 (2004) [hep-ph/0309342].
- [26] A. Pilaftsis, Phys. Rev. D **56**, 5431 (1997) [hep-ph/9707235].
- [27] B. Adhikary, A. Ghosal and P. Roy, JCAP **1101**, 025 (2011) [arXiv:1009.2635 [hep-ph]].
- [28] M. A. Luty, Phys. Rev. **D45** (1992) 455.
- [29] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, U.S.A) (1990).
- [30] B. Adhikary, Phys. Rev. D **74**, 033002 (2006) [hep-ph/0604009].
- [31] A. Abada, S. Davidson, A. Ibarra, F. -X. Josse-Michaux, M. Losada and A. Riotto, JHEP **0609** (2006) 010 [hep-ph/0605281].
- [32] S. Antusch, S. F. King and A. Riotto, JCAP **0611**, 011 (2006) [hep-ph/0609038].
- [33] J A. Harvey, M S. Turner, Phys. Rev. D **42**, 3344 (1990)
- [34] E. Giusarma, R. de Putter, S. Ho and O. Mena, Phys. Rev. D **88**, no. 6, 063515 (2013) arXiv:1306.5544 [astro-ph.CO].
- [35] C. L. Bennett *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **208**, 20 (2013) [arXiv:1212.5225 [astro-ph.CO]].
- [36] H. Aihara *et al.* [SDSS Collaboration], Astrophys. J. Suppl. **193**, 29 (2011) [Erratum-ibid. **195**, 26 (2011)] [arXiv:1101.1559 [astro-ph.IM]].
- [37] M. Auger *et al.* [EXO Collaboration], Phys. Rev. Lett. **109** (2012) 032505 [arXiv:1205.5608 [hep-ex]].
- [38] A. Giuliani, Acta Phys. Polon. **B 41** (2010) 1447.
- [39] W. Rodejohann, J. Phys. **G 39** (2012) 124008 [arXiv:1206.2560 [hep-ph]].
- [40] A. Ghosal, Y. Koide and H. Fusaoka, Phys. Rev. D **64**, 053012 (2001) [hep-ph/0104104].